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THE COEFFICIENTS OF GAS VISCOSITY. II.

By Willard J. Fisher.

Construction of Apparatus.

THE apparatus described in the following pages was constructed with the facilities and help of the department of physics, Cornell University, and, toward the end of the construction, with the aid of a grant from the Rumford Fund by the American Academy of Sciences, in carrying out a research on the viscosity of gases and its variation with the temperature.

In a previous paper,[1] Part I. of this series, the writer has shown how Sutherland's equation for gas viscosity may be represented by a straight line, and has used the method for finding the two constants of the equation, taking as a basis the published data of von Obermayer, Breitenbach, Schultze, Markowski, E. Wiedemann, Holman and Bestelmeyer. Since the appearance of Part I. Grindley and Gibson[2] have made researches on the flow of air through long metal tubes, and in reducing their results used the same graphical method. All the data published lead to the conclusion that Sutherland's equation accurately represents the facts for temperatures above the critical and pressures such that Boyle's law holds. The graphical representation of the data in Part I. led the writer to look for relations between molecular weight, ratio of specific heats and other constants of pure gases, and their viscosity constants. Certain indications of relations were found, but the

[1] Willard J. Fisher, Phys. Rev., 24, p. 385, 1907.

[2] J. H. Grindley and A. H. Gibson, Proc. Royal Soc., A, 80, p. 114, 1908.

experimental basis is as yet too narrow for definite conclusions. It is hoped that the apparatus here described will ultimately throw light on this problem, whose solution will undoubtedly bear on that of the partition of energy.

Apparatus — General. — The general idea of the apparatus was suggested by that made at Halle by H. Schultze,[1] and afterward used by H. Markowski, F. Kleint and P. Tänzler. The advantages of Schultze's arrangement are considerable — it has entire symmetry about the capillary and uses but a small quantity of gas, which is forced back and forth through the capillary, and so allows of an indefinite number of determinations with identically the same portion of gas. Hence, also, great pains can be taken to secure purity of the gas used.

In the apparatus to be described all pressures were measured with mercury, and a zero manometer — a barometer, but dipping into the same reservoir of mercury as the manometers for measuring difference of pressure — was made an integral part of the apparatus; this enables the pressures on the two sides of the capillary to be measured with the same steel scale, instead of using two separate scales and a separate barometer. Schultze's water manometer method is no doubt more sensitive, but is also more complicated in construction and use. The gas pipettes and mercury buckets for measuring the volume of gas and securing a constant pressure were all made cylindrical, which allows easier construction and a more nearly automatic method of pressure regulation by an electric windlass; the gas pipettes were calibrated and mounted in such a way that frequent observations of time and volume could be made, to eliminate accidental errors of observation. The capillary was made of Jena 59 III glass, on account of its high melting point and excellent qualities as regards after-working and low expansion coefficient; it was also calibrated by a graphical method, considered to be an improvement on Schultze's, as it permits very numerous observations to be utilized in the calibration. For varying the temperature of the capillary an electric furnace was used, and for measuring its mean temperature a platinum thermometer with compensated leads after Callendar's plan, which plan was also used in the construction of the bridge.

[1] H. Schultze, Ann. d. Phys., 5, p. 140, 1901.

Manometers. — The glass parts of the pressure measuring apparatus and connections for filling are shown in Fig. 1, together with the gas pipettes and the water jackets for maintaining a reasonably constant temperature in the pipettes.

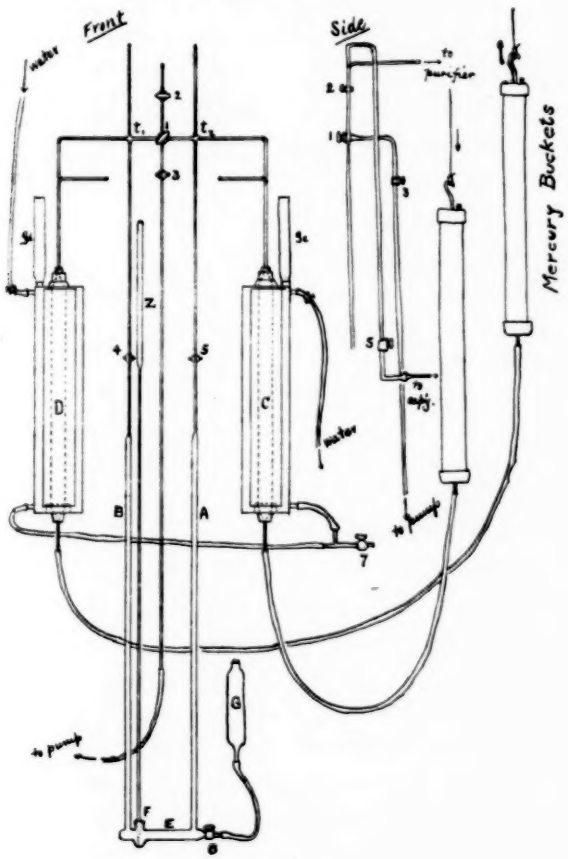

Fig. 1.

A and *B* are two glass tubes of about 1 cm. bore, 84 cm. long, 12 cm. apart, joined below by a horizontal tube *E*, 2 cm. diameter, and connected above to two four-way joints t_1 and t_2. At *F* there is a cup-shaped opening, and into this there is ground in and cemented tight a tube 94 cm. long ; this ends below in a prolongation with a small opening, which dips into a cup formed on the under side of *E*, and above in a tube *Z* about 1 cm. bore and 30

cm. length, sealed shut at its upper end. The tubes A and B are the pressure manometers, and by their readings the difference of pressures on the two sides is determined; Z is exhausted of air and inverted in the mercury of E, and serves as a zero manometer; by the difference between its reading and those of A and B the true pressures in the latter are determined. By raising and lowering G the mercury levels in A, B and Z are adjusted.

The four-way joints t_1 and t_2 receive the tubes from A and B before described, and send branches sideways to the gas pipettes through glass springs, upwards to tubes which bend over downward behind the apparatus, join the stopcocks *4* and *5*, and then turn backwards to the capillary. Inwards branches go from t_1 and t_2 to a central four-way cock *1*. From this a branch goes upward through stopcock *2*, and bends backward to the gas-generator and purifier; also a branch goes straight backwards from *1* and bends downward through cock *3* to the air-pump hose. All tubing not otherwise specified is of about 2.5 mm. bore.

With the exception of one well shellacked short rubber connection between cocks *1* and *3*, and of ground joints at F and between cocks *4*, *5* and the capillary, all made tight with De Khotinsky cement, the joining of glass tubing was done with the blowpipe, followed by as careful annealing as possible.

Manometers A and B are not of exactly the same diameter, but fortunately the piece of tube of which the wide part Z of the zero manometer was made was found to give exactly the same capillary depression of mercury as tube B; this was determined by inserting the tube in F, and comparing the levels.

The zero manometer was made with the prolonged lower end closed and bent over at the tip, and was exhausted through the upper end of Z while sealed, in a horizontal position, to a Sprengel pump. During the exhaustion the air film on the walls was removed by heating as hot as it was considered safe to do with a Bunsen burner; then the end of Z was sealed off from the pump, a little nick scratched just above the turned over end of the prolongation, this dipped below the mercury at F, and the tip knocked off. The sealing of the joint with De Khotinsky cement was then completed at leisure.

All stopcocks were made tight with a grease recommended by Prof. A. W. Browne, of the department of chemistry: four parts of vaseline and one of paraffine were melted together, and the mixture filtered hot. This proved very efficient in making tight joints, and has the advantage of being chemically inactive.

The barometer used in checking the zero manometer was by Henry J. Green, of the Fortin type, with brass scale; the internal diameter of the tube is not known, but is estimated to be 1.5 cm.

A portion of the steel scale, with its sliders, is shown in Fig. 2.

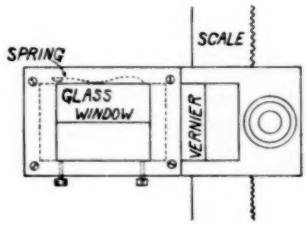

Fig. 2a.

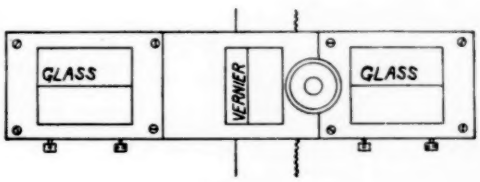

Fig. 2b.

It was made by Brown and Sharpe Mfg. Co., and is 1 inch wide, 3/8 inch thick and 127 cm. long, with graduation to millimeters. It is guaranteed accurate at 62° F., which was justified by comparison with a Geneva Society standard meter bar. On its right edge a rack is cut, into which the pinions of the sliders mesh. The sliders are of brass; Fig. 2*a* shows that used with the zero manometer, Fig. 2*b* one of the two made for the pressure manometers. Either may be moved to any position by a rack and pinion. The bearing surface is on the long straight left side of the scale, while strong spring plungers press against the right side to give accurate travel. An opening is out in front to the graduations, and contains a vernier on silver reading to .05 mm. Each slider carries one or two glass windows, with hair lines, adjustable upward by capstan-head screws, pressed against these by springs, and held against the brass frames by felt packing. The hair lines were adjusted in line with the vernier zeros and perpendicular to the bar by the help of the sliding bed and microscopes of a small Geneva Society dividing engine. The vernier zero is placed at the center for convenience in reading the scale. Parallax was eliminated by reflection from

mirrors fastened to the supporting board behind the mercury columns. The hair line was brought *down* to apparent contact with the top of the meniscus. The mercury tubes were tapped before reading, to insure constant form of the meniscus.

The scale was adjusted to the vertical with a reversible level; the vernier readings were checked against each other by measurements of the same height; the parallax mirrors were tested for curvature and inclination with an accurately horizontal telescope provided with a vertical scale and long spirit level — the reflected image of the scale, seen against the cross-hairs, enabled a calculation of the error to be made. The two pressure manometers were also compared, to secure corrections for capillary depression. A table of values was computed for the correction factor to reduce observed heights of mercury to true centimeters at 0° C., which covered the range 10° to 30° C., and was applied to all observations. The zero and left-hand manometers, used as a siphon barometer, were compared with the Green barometer, and found to agree with this as well as its somewhat doubtful capillary correction would allow.

Gas Pipettes and Water Jackets. — The original five-foot length of glass tubing from which the pipettes were made was selected from stock, and was very nearly 2.8 cm. in internal diameter at each end and 1.7 mm. wall thickness. The pipettes were about 44 cm. long, and graduated in centimeter divisions with a beam compass, using the same Brown and Sharpe steel scale (marked *a*) which was used in the calibration of the capillary tube. The graduations were etched in with hydrofluoric acid. The outsides of the pipettes were silvered by Brashear's method, and a strip down the back coated with shellac, then the rest of the silver removed. The graduations were blackened with grease pencil. The construction gives an inverted real image of a graduation-mark, distant from the back two thirds the radius of the tube. By placing the eye in such a position that the tip of a graduation line is in apparent contact with the tip of its image a very efficient correction for parallax is obtained. The pipettes were set in the water jackets, cleaned with sodium hydrate, chromic acid and distilled water, stopcocks sealed on below and the upper ends sealed by glass spring tubes to the manometers.

The water jackets were made of sheet copper, each with tubes on the side above and below for admission and exit of water, an opening centrally below for a stopper, with inside a larger circular rim about it for cement; above two openings, one central, with a screw cap (perforated) for a stopper, another beside it, to hold the gauge tube g and thermometer. The pipettes, the glass front windows and the gauge tubes were set with glycerine and litharge cement.

The two water jackets were connected below by rubber tubing, with a T-joint and stopcock 7 for emptying, when necessary. Above, each was connected by a flexible tube to a large water bucket tapped near its bottom. Raising one bucket and lowering the other caused water to flow steadily through the jackets, the rate being controlled by pinch cocks. The glass gauge tubes g showed by their difference of water-level whether water was flowing or not, and also served for the introduction of the thermometers. Since the water supply was thus quite large in quantity, and at the temperature of the room, the temperature of the pipettes could be maintained nearly constant for quite long times.

After setting the parts, allowing the cement to harden, and connecting the pipettes to the manometers, the whole apparatus was exhausted and let stand, to test the tightness of the blown glass joints and the stopcocks. Then the pipettes were filled with mercury and calibrated.

The calibration of the pipettes was carried out according to the following theory.

The volume between the top and any cross-section of a right circular cone is expressed by

$$V = x(a + bx + cx^2),$$

in which x is the distance from the top of the frustum to the section in question, and a, b, c are functions of the base radii and total height of the frustum. Since an ordinary glass tube is somewhat conical, an expression of the same form may express the volume between graduation marks; then x represents the distance between an assumed zero graduation and any other. Volume calibration of a glass tube graduated in centimeters, like one of these pipettes,

consists then in finding the values of a, b, c; these are not to be supposed equal to the coefficients of the right circular frustum, as the elements of the bounding surface are probably not straight lines.

If the length be divided into five parts, whose ends are designated by $x_0 \cdots x_5$, then mercury may be run out and weighed and the volume determined from the observed temperature; starting from the point x_0 and running out the portion from x_0 to x_1, then in addition *into the same bottle* the portion from x_1 to x_2, etc., then the masses m_1 (from x_0 to x_1), m_2 (from x_0 to x_2), etc., are known. $m_3 - m_0$ is the mass from x_3 to x_0, $m_4 - m_1$ the mass from x_4 to x_1, $m_5 - m_2$ that from x_5 to x_2; these masses are nearly equal and determined by independent weighings, while $x_3 - x_0$, $x_4 - x_1$, $x_5 - x_2$, are equal if the graduations are good. Call their common value d, and the density of mercury ρ, constant if the temperature control is good. Then

$$m_0/\rho = x_0(a + bx_0 + cx_0^2), \tag{1}$$

and five similar equations with different subscripts. Hence

$$\frac{m_3 - m_0}{\rho(x_3 - x_0)} = \frac{m_3 - m_0}{\rho d} = a + b(x_3 + x_0) + c(d^2 + 3x_3x_0) \tag{2}$$

and two like equations with subscripts (4, 1), (5, 2). From these last three equations a, b, c are found.

The equations have been written in the above form for symmetry and hence brevity; but in using them x_0 and m_0 must be taken equal to zero — the calibration must begin with the zero of the graduation. The differences $x_1 - x_0$, $x_2 - x_1$, . . . , need not be equal; then the computations are slightly longer.

TABLE I.

Pipette.	Mean Temperature.	a	b	c
C	21.70°	5.4750	−0.0005	$-1.27 \cdot 10^{-5}$
D	21.38	5.3768	$2.47 \cdot 10^{-3}$	$3.0 \cdot 10^{-6}$

Three sets of weighings were made for pipette D; from them the calibration calculations were carried out, the results forming Table I.

Pipette C was not so accurately calibrated, owing to trouble with the silver coating, and was not often used in runs.

These constants give check values for both pipettes agreeing with the mean weighings much better than the observed weights agree among themselves.

The balance used was a lecture balance by Ruprecht, sensitive to less than .05 gram with over 3 kg. each side. The weights were corrected to .10 gram by standards. Double weighing was used.

For determining the temperature of the pipettes three thermometers were calibrated, Baudin's make, numbers 8,536 (range $-1.5°$ to $30°$), 12,979 ($-2°$ to $43°$), 15,000 ($-1°$ to $41°$), all divided to $\frac{1}{10}°$. These were tested in melting ice, with the following results, estimating hundredths.

8,536	read	+0.10.	May 15, 1908.	Thermometer dated Sept., 1880.
12,979	"	+0.00.	May 11, 1906.	" " June, 1891.
"	"	+0.00.	Jan. 15, 1907.	
"	"	+0.01.	May 30, 1908.	
15,000	"	+0.02.	May 11, 1906.	" " July, 1899.
"	"	+0.03.	Jan. 15, 1907.	
"	"	+0.03.	May 30, 1908.	

The standards for comparison were two sensitive Baudin thermometers, graduated to .02°, which had been calibrated at the Bureau of Standards for the work done by Mr. F. E. Kester in this laboratory on the Joule-Thomson effect. Their ice points were tested by the writer, for comparison with the Bureau of Standards' determinations.

Baudin No.	Bureau Standards.			W. J. F.	
	Number.	Ice.	Date.	Ice.	Date.
10213	945	0.4206	Mar. 10, 1905	0.420	Dec. 14, 1907
10214	946	0.4600	Mar. 10, 1905	0.460	Dec. 14, 1907

The apparatus for comparison was made of an unsilvered Dewar cylinder large enough to contain all the thermometers to be compared and supplied with a heating coil inside at the bottom, and a tube through which air could be forced to act as a stirrer, which also carried supports for the thermometers. The amount of water used was sufficient for complete immersion of the stems, as this was the way they were to be used in the water jackets. The errors of the three thermometers were determined over a range from 12° to 27°;

and smooth correction curves plotted. The ranges of these corrections were from —.10° to —.15° for B. 15,000; —.05° to —.10° for B. 12,979; —.14° to —.18° for B. 8,536.

This determination of errors is believed to be accurate within $\frac{1}{40}°$.

For exhausting the apparatus there was used an oil pump, connected by a short rubber hose and long glass tube to cocks *3* and *1*. The vacuum obtained was tested by a small mercury gauge, placed near the purifying apparatus and connected by glass tubing and blown joints to cock *2*. The mercury in this gauge was very thoroughly cleaned, and boiled to expel air. Before connecting the mercury buckets and hose, while the lower ends of the pipettes were still closed by stopcocks, the air was pumped out to a 5.5 mm. vacuum and the apparatus let stand for sixteen hours; the whole interior formed one vacuum, the external air being excluded by nine stopcocks. At the end of the time no leakage could be detected by the gauge.

The whole apparatus was supported on a wooden frame, except the electric furnace, which stood on a table screwed to the floor behind the frame. The frame was screwed to the timbers of the ceiling and supported on the floor by small screw-jacks. All parts were fastened together with screws and bolts, and those parts which immediately supported apparatus were made adjustable in two horizontal directions.

Pressure Regulation. — The mercury buckets were made each of a piece of steam pipe, 20 inches long, 1¼-inch bore, with screw caps on each end. The upper cap was fitted with a hook and bored on one side of the center with a tapered hole closed with a tapered screw plug. The lower cap was bored for the end of a taper-threaded small iron pipe, for the attachment of the hose for connection to the gas pipettes. After assembling the buckets were cleaned with gasoline, dried, shellacked inside and the joints made tight with shellac; the whole, especially the joints, was then heated hot enough to get rid of the solvent of the shellac. The buckets were suspended from the windlass by the heaviest obtainable tinned braided picture cord.

To test the buckets for tightness, before filling with mercury the

screw plugs were cemented tight, the whole apparatus exhausted to about 6 mm., and let stand. There was a slow leakage shown, due no doubt to passage of air through the rubber tubing or perhaps to escape of air from its walls.

To make sure that this leakage should do no harm by contaminating the pure air or other gas contained in the pipettes, the necessary amount of mercury was put into the buckets and the screw plugs cemented tight. The buckets were laid on the floor, so that all mercury ran down into them, and the whole apparatus exhausted as far as the slow leakage of the pressure tubing and the pump would allow — about 6 mm. The buckets were then raised to allow the mercury to run into the pipettes. Care was thereafter taken to prevent the mercury surface from running down into the hose, so that it was always in contact with glass.

In a similar way, by closing the pinchcock below *G*, opening *6* and exhausting, air is removed from the rubber hose which admits mercury to the manometers.

For raising and lowering the buckets an electric windlass was constructed with a $\frac{1}{16}$ h.p. shunt motor belted to a pulley, giving a speed reduction of $\frac{1}{2}$, and two worms and gears in series, giving reductions of $\frac{1}{61}$ and $\frac{1}{99}$ respectively; the resulting reduction was then $\frac{1}{12078}$. The motor was run separately excited, the field was excited with 110 volts, the armature shunted off a Cutler-Hammer rheostat, with an additional step formed of a tin frame. The range of adjustment was about 100 to 1,200 revolutions per minute. The picture cord was wound up on small iron pulleys of various sizes. The windlass was supported on a wooden pillar resting on small screw-jacks, by means of a wooden frame made exactly adjustable as to height.

All mercury used in the apparatus was cleaned by falling in excessively fine drops a distance of about four feet through a solution of nitric acid and mercurous nitrate, then passing through a goose-neck into concentrated sulphuric acid, then out into a bottle through another goose-neck. This was repeated several times and seems to give very pure mercury; for that used in the vacuum gauge, which had been treated this way more frequently than most of the rest, showed a perfectly flat surface on the vacuum side.

Fig. 3 shows in diagram the essential features of the apparatus for regulating pressure on the two sides of the capillary: l_1 is the height of the bottom of one of the mercury buckets from the floor; H_1 the height of the mercury in this bucket above the bottom; h_1 the height of the mercury level in the corresponding pipette from the floor; m_1, p_1 and v_1 the mass, pressure and volume of the gas

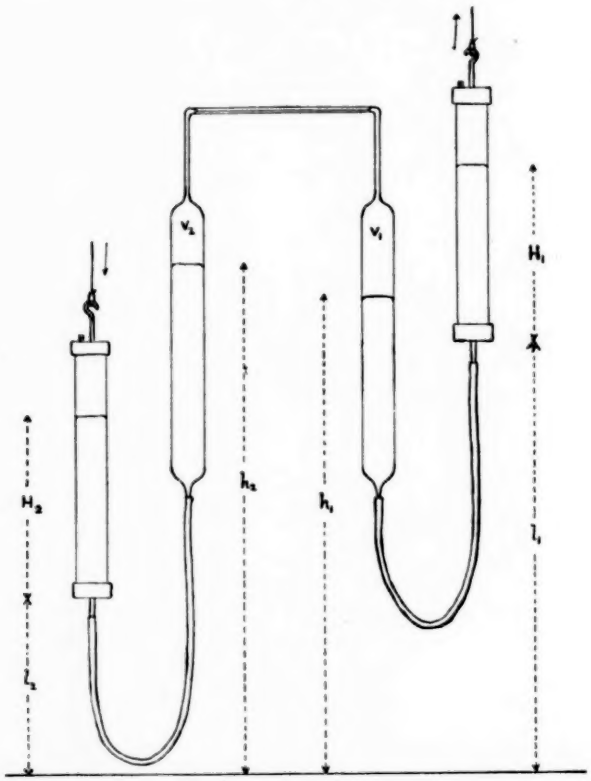

Fig. 3.

inclosed; Q_1 the cross-section of the cylindrical bucket; q_1 the cross-section of the pipette; ρ the density of mercury; B the barometric pressure; similarly, with subscript $_2$ for the other side of the apparatus. The condition for constant pressures on the two sides may be approximately found as follows:

By Boyle's Law, $p_1 v_1 = cm_1$; since p_1 is constant, any variation will give

$$p_1 \delta v_1 = c \delta m_1.$$

By the hydrostatic law, $p_1 + B + \rho g (l_1 + H_1 - h_1) = 0$; whence

$$\delta l_1 + \delta H_1 - \delta h_1 = 0.$$

Since mercury is not compressible, and pressure tubing does not stretch,

$$Q_1 \delta H_1 + q_1 \delta h_1 = 0.$$

Since glass tubing is inextensible

$$\delta v_1 + q_1 \delta h_1 = 0.$$

Combining these and a similar set for the other side,

$$\left.\begin{aligned} \delta l_1 + \frac{c \delta m_1}{p_1}\left(\frac{1}{q_1} + \frac{1}{Q_1}\right) &= 0, \\ \delta l_2 + \frac{c \delta m_2}{p_2}\left(\frac{1}{q_2} + \frac{1}{Q_2}\right) &= 0. \end{aligned}\right\}$$

In addition there are as connecting conditions, that the mass of gas entering the capillary is the same as that leaving it, whence

$$\delta m_1 + \delta m_2 = 0$$

and that the velocity ratio of the two buckets is constant, owing to the construction of the apparatus, whence

$$\delta l_1 + k \delta l_2 = 0.$$

Reducing these and solving for k,

$$k = \frac{p_2}{p_1} \frac{\frac{1}{q_1} + \frac{1}{Q_1}}{\frac{1}{q_2} + \frac{1}{Q_2}}. \tag{3}$$

If $q_1 = q_2$ and $Q_1 = Q_2$, then $k = p_2/p_1$. So that by careful selection of the glass for the pipettes and use of cylindrical mercury buckets of the same size, moving with a constant velocity ratio, it is possible to secure constancy of pressure. This condition, expressed in equation (8), is necessary, but not sufficient; for the speed at which the apparatus for raising one bucket and lowering the other must be run depends upon the particular ratio of pressures selected, on the values of those pressures, and on the speed with which the gas under

test passes through the capillary. Trial with air has shown that the condition derived above gives a good clue for starting on trials, and that it is not difficult to find two pressures and a corresponding speed such that the pressures remain constant almost without attention to the motor-rheostat for quite long times.

Computation of the Flow of Gas. — In making runs the instant of passage of the mercury surface by a graduation mark in the pipette was observed for as many consecutive graduations as possible. The time was given by the click of an electrically operated clock, which received an impulse every other second from the Howard standard clock of the laboratory. These observations were computed by subtracting from the volume (reckoned from the zero of the graduation) represented by the last graduation mark observed the volume represented by the first; also the last observed time from the first; this was done again with the next to the last and the second graduation marks and times, and so on for all the observed pairs. The sum of these volume differences divided by the sum of the time differences gives the mean rate of flow of gas In symbols for one pair of observations the constant flow

$$F = \frac{v'' - v'}{t'' - t'}$$

and for any number of pairs

$$F = \frac{\Sigma(v'' - v')}{\Sigma(t'' - t')} = \frac{\Sigma(v'' - v')}{\Sigma(t'') - \Sigma(t')}. \qquad (4)$$

In this $\Sigma(t')$ represents the sum of the first half of the observed times, $\Sigma(t'')$ the sum of the last half.

Since the volumes to any two graduation marks x'' and x' may be represented by

$$v'' = x''(a + bx'' + cx''^2)$$

and

$$v' = x'(a + bx' + cx'^2);$$

therefore

$$v'' - v' = (x'' - x')[a + b(x'' + x') + c(x''^2 + x''x' + x'^2)],$$

and this relation may be used to shorten the tedious summations indicated above.

The graduations being supposed accurate centimeters, and ob-

servations being taken on consecutive graduation marks, the pairs of graduations first and last, second and next to last, etc., have a common mean, which may be represented by x_m. For any pair x'', x',

$$x'' - x_m = x_m - x' = \Delta,$$

say. Then

$$v'' - v' = 2\Delta[a + 2bx_m + 3cx_m^2 + c\Delta^2].$$

Putting $a + 2bx_m + 3cx_m^2 = k$,

$$v'' - v' = 2[k\Delta + c\Delta^3].$$

For all the observations

$$\Sigma(v'' - v') = 2k\Sigma\Delta + 2c\Sigma\Delta^3.$$

If the number N of observations is odd, Δ changes by unit steps from 1 to $\frac{1}{2}(N-1)$, and

$$\Sigma(v'' - v') = \tfrac{1}{4}(N^2 - 1)\left[k + \frac{c}{8}(N^2 - 1)\right]. \qquad (5)$$

If N is even, Δ changes by unit steps from $\frac{1}{2}$ to $\frac{1}{2}(N-1)$, and

$$\Sigma(v'' - v') = \frac{N^2}{4}\left[k + \frac{c}{8}(N^2 - 2)\right]. \qquad 6)$$

These two formulas enable the computation of a large number of observations to be made with comparative ease.

Capillary Construction and Calibration. — The capillary tube was of Jena 59 III glass, selected by examination of the ends for roundness of the bore and suitable size. A piece of about 50 cm. length was cut, and its ends sealed with the help of a considerable air pressure to short pieces of the same material but internal diameter about 2 mm. and length about 10 or 15 cm. The cleaning in preparation for calibration was carried out with caustic soda or potash, aqua regia, potassium bichromate and sulphuric acid, with water redistilled in glass for washing. The solutions were forced through the tube hot, by inclosing the tube to be cleaned in a glass jacket like a Liebig condenser into which steam was continually passed. The apparatus stood upright, with the lower end of the wide connecting tube passing from the steam bath through a stopper into a side-neck test-tube, where it dipped below the surface of a quantity of the cleaning solution. Air pressure from a reservoir

introduced by the side neck forced the liquid up through the hot capillary. After such cleaning and rinsing with distilled water each end tube was plugged with cotton wool (absorbent cotton) and air from a sulphuric acid drying tube drawn through until no water drops appeared. The tube was examined with a magnifier, and if any discoloration or suspicious spots appeared the cleaning was repeated.

The tube was calibrated by a method suggested by that described by H. Schultze,[1] but differing from his in the use of a graphical reduction and a larger number of measurements of the mercury thread.

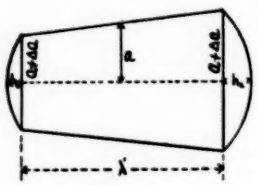

Fig. 4.

In a good capillary tube an inclosed short mercury thread may be taken to have the form of the frustum of a right circular cone, capped at each end with a spherical segment of one base (Fig. 4).

Taking a the arithmetical mean radius of the frustum, $a - \Delta a$ and $a + \Delta a$ the radii of its two end faces, λ' its height, h_1 and h_2 the heights of its smaller and larger segments, then its volume is given by

$$V = \pi a^2 \left\{ \left(1 + \frac{\Delta a^2}{3a^2} \right) \lambda' - \frac{\Delta a}{a} (h_1 - h_2) + \left(1 + \frac{\Delta a^2}{a^2} \right) \frac{h_1 + h_2}{2} + \frac{h_1^3 + h_2^3}{6a^2} \right\}. \tag{7}$$

In practice $\Delta a/a$, h_1^3, h_2^3 and $(h_1 - h_2)$ are small in comparison with λ', and may be neglected, when the formula becomes

$$V = \pi a^2 \lambda' + \frac{\pi a^2}{2} (h_1 + h_2).$$

Assuming the volume of an equivalent cylinder $V = \pi a^2 \lambda$ we have

$$\lambda = \lambda' + \tfrac{1}{2}(h_1 + h_2) \tag{8}$$

as its length. This equivalent length of the mercury thread was used in the calibration.

The mean radius a is situated nearly midway between the ends of the mercury thread, with a negligible relative error, practically less than $\frac{1}{6} \Delta a/a$.

[1] H. Schultze, Ann. d. Physik, 5, p. 147, 1901.

The calibration is performed in two steps, of which the following is the theory:

1. *For Volumes* (Fig. 5). — From any arbitrary position (in practice marked by etching the tube) assumed as zero, measure the position l of the center of a short mercury thread, and the length λ of the thread.

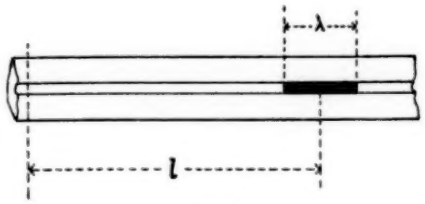

Fig. 5.

At the point l the cross-section of the tube is $Q = V/\lambda$, in which V is the volume of the thread. Measured from zero the total volume of the tube to l is v. Also, $dv = Q dl = V/\lambda \cdot dl$. If V is kept constant by temperature control,

$$v = V \int_0^l \frac{dl}{\lambda}.$$

Plotting a curve with $y = 1/\lambda$, $x = l$, (Fig. 6, I.), its area

$$= \int y dx = \int_0^l \frac{dl}{\lambda} = \frac{v}{V}.$$

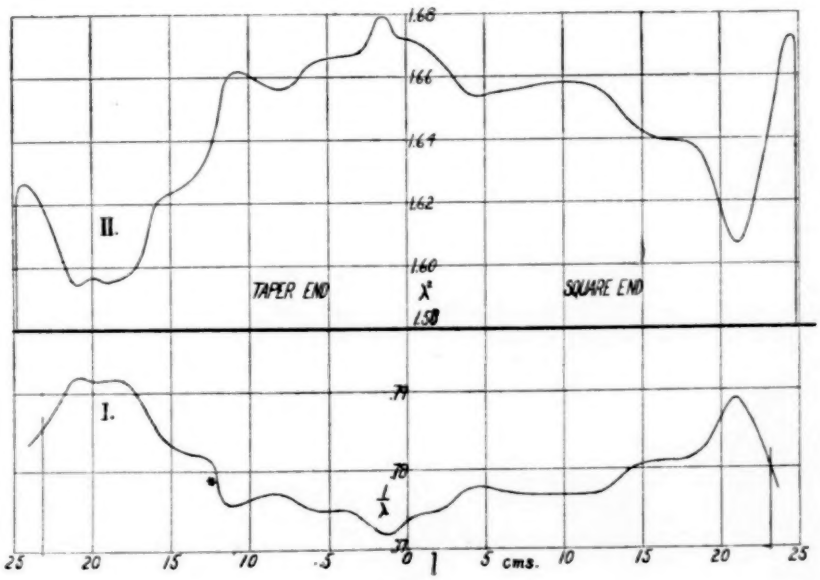

Fig. 6.

The volume V of the short thread cannot be computed accurately by direct weighing, since the container must be of much greater mass than the mercury itself. Hence a more accurate way to find V is to find the total volume of the tube by a *long* thread between two points l' and l''. Call this volume $v'' - v'$. Then

$$V = \frac{v'' - v'}{\int_{l'}^{l''} \frac{dl}{\lambda}}, \tag{9}$$

in which the integral is the area under the curve already drawn and between the ordinates at l' and l'', so that V is determined in terms of measured quantities.

2. *For Viscous Resistance.* — The equation for the flow of a gas through a tube of truly cylindrical bore is (omitting the very small correction $4\zeta/a$)

$$F = \frac{\pi a^4}{16 p \eta' l}(p_1^2 - p_2^2) \quad \text{or} \quad \frac{16 p F \eta'}{\pi} = \frac{a^4}{l}(p_1^2 - p_2^2)$$

(O. E. Meyer, Pogg. Ann., 127, p. 269, 1866; Poynting and Thomson, Properties of Matter, p. 210).

In this F = the volume transpired per second, measured at pressure p;

a = the radius, l = the length of the tube;

p_1 = the entrance, p_2 = the exit pressure.

If this law is assumed true for small lengths, such as the gas may be considered to flow through in series in a tube of slightly varying bore, then for each such tube of radius a and length dl.

$$\frac{16 p F \eta'}{\pi} = a^4 \frac{d(p^2)}{dl}.$$

In this the left-hand member is constant. So

$$d(p^2) = \frac{16 p F \eta'}{\pi} \frac{dl}{a^4}$$

and

$$\int_0^l d(p^2) = p_1^2 - p_2^2 = \frac{16 p F \eta'}{\pi} \int_0^l \frac{dl}{a^4}.$$

Now, from the previous curve,

$$Q = \pi a^2 = \frac{V}{\lambda}; \quad \therefore \frac{1}{a^4} = \frac{\pi^2 \lambda^2}{V^2}; \quad \therefore p_1^2 - p_2^2 = 16 p F \eta' \frac{\pi}{V^2} \int_0^l \lambda^2 dl. \quad (10)$$

Graphical evaluation of $\int_0^l \lambda^2 dl$, with the use of the previously determined factor V, gives the calibration factor depending on the shape of the tube. Hence

$$\eta' = \frac{V^2(p_1^2 - p_2^2)}{16\pi p F \int_0^l \lambda^2 dl} = \frac{(v'' - v')^2 (p_1^2 - p_2^2)}{16\pi p F \int_0^l \lambda^2 dl \left[\int_v^{l''} \frac{dl}{\lambda}\right]^2}. \quad (11)$$

(This equation shows that small errors in λ affect the values of the two integral expressions nearly equally but *oppositely*, and so produce an effect on the whole proportional to their difference, and nearly negligible.)

O. E. Meyer[1] shows by discussion of the results of Kundt and Warburg, Warburg and Breitenbach, that ζ is numerically equal to the mean free path of the molecule, and like it varies inversely as the pressure. Also, in his approximate solution of the equations of the gas capillary[2] he makes fundamental assumptions which may be shown to lead to the equation

$$\left(\frac{r^2 - a^2}{4} - \frac{\zeta a}{2}\right) \frac{\partial p}{\partial x} = \eta u,$$

in which a is the radius of the tube, r the distance of an element of the fluid from the axis, p the pressure, u the velocity, and x distance parallel to the axis. Assuming $\zeta = k/p$, and integrating, the equation for the time rate of discharge is found to be

$$F = \frac{\pi a^4 (p_1^2 - p_2^2)}{16 \eta p l} \left[1 + \frac{4}{a} \frac{k}{\frac{p_1 + p_2}{2}}\right], \quad (12)$$

$k/\frac{1}{2}(p_1 + p_2)$ is ζ computed for the arithmetic mean of the entrance and exit pressures.

From the observations η' is computed as above. Using this, ζ is computed from the Kinetic Theory equations for mean free path,

[1] Kinetic Theory of Gases, R. E. Baynes, Trans., p. 211.

[2] O. E. Meyer, Pogg. Ann., 127, p. 268, 1866.

$\eta' = \frac{1}{8}\rho GL$, $p = \frac{1}{2}\rho G^2$, with $p = (p_1 + p_2)/2$; a is the *mean* radius of the tube. Thence the correction term $4\zeta/a$ is applied to give the true viscosity coefficient;

$$\eta = \eta' \left(1 + \frac{4\zeta}{a}\right). \tag{13}$$

The capillary was marked near its middle with a short scratch etched in with hydrofluoric acid, and this mark used as a reference point. The tube and a steel centimeter scale were clamped to the sliding plate of the dividing engine, and a pointer fastened to its bed, so that distances moved by the plate and the tube could be read, estimating tenths of millimeters by the scale and pointer. The reference mark was brought under the cross hairs, the steel scale read, and the mercury thread brought by air pressure so that one end appeared in the field. The dividing engine screw was turned to bring the edge of the meniscus under the cross, the steel scale and the head of the engine screw were read, the latter to hundredths of a millimeter, and the height of the meniscus was found by two settings with the eyepiece micrometer. This was repeated with the other end of the thread, which was then moved to a new position and the measurements repeated. The thread was forced through the tube several times, and in all 105 measurements of the thread were made.

A Baudin 40° thermometer, No. 12,979, whose bulb was fastened opposite the microscope, was used to find if rapid changes in temperature of the air had any perceptible effect on the length of the mercury thread. On the conclusion being formed that there was no such effect, the thermometer was read only to tenths, and the more gradual changes of the room temperature were observed and corrected for, reducing the thread length to a standard temperature. Calling λ_2 the thread length obtained by adding to the length of the frustum the half sum of the two meniscus-heights, λ_1 the thread-length at standard temperature, t_2 the room temperature, t_1 the standard temperature, α the cupical expansion-coefficient of mercury $= 0.0001802$, β the linear expansion-coefficient of the glass of the tube $= .0000059$, and neglecting squares and products of small quantities, then

$$\frac{\lambda_1}{\lambda_2} = 1 + (\alpha - 2\beta)(t_1 - t_2) = 1 + 0.0001684\,(t_1 - t_2).$$

The thread of the steel screw of the dividing engine and the graduation of the steel scale (one made by Darling, Brown & Sharpe) were also compared, and found to agree closely enough for the purpose of this experiment on the supposition that the screw was cut at 0° C., the scale graduated at 62° F. (which is what the makers advertise). It was found that

$$\frac{\text{scale-readings}}{\text{screw-readings}} = 1.00023.$$

A correction for the temperature at which the measurements were made was calculated and applied, reducing them to the centimeter of the D., B. & S. scale. This scale was compared with a Geneva Society standard meter belonging to the Department of Physics, and was found to be correct.

The measurement of the tube could not be carried out by means of the mercury thread clear to the flaring ends, where the small tube was sealed to the larger heating tubes, so that in these regions recourse was taken to micrometric measurements. The intention was to measure the apparent diameter of the bore of the tube, as seen through the glass, at points whose positions with regard to the zero were also to be measured, and to reduce these to real diameters and equivalent thread-lengths; but the difficulty of obtaining sufficiently concordant micrometer readings was found to be so great that the reduction to thread-lengths was abandoned. Instead, the apparent diameters were measured and from them it was found at what point they became so large that their effect on the resistance factor could safely be neglected. The two points so found were assumed as the ends of the tube. The uncertainty due to this method affects only a few millimeters at each end of the tube, as the flare was very rapid.

Table II. gives the values of l, distance from the reference mark to the middle of the thread, and λ, length of thread, corrected for screw error, temperature and meniscus; the standard temperature assumed was 21°.2 C. The measurements are grouped as made in runs, and the direction from the reference mark is indicated by

TABLE II.

CAPILLARY $J_3{}^a$.

Run.	l	λ	Run.	l	λ	Run.	l	λ
I.	0.295	1.2930	III.	3.48	1.2910	V.	13.12	1.2859
Taper.	1.165	1.2957	Taper.	4.65	1.2910	Square.	14.84	1.2817
	4.38	1.2902		6.405	1.2900		16.87	1.2805
	7.965	1.2867		8.705	1.2885		18.38	1.2801
	9.22	1.2880		9.90	1.2870		19.80	1.2735
	10.625	1.2890		11.295	1.2892		21.15	1.2673
	12.45	1.2796		12.55	1.2793		23.24	1.2830
	13.82	1.2762		14.88	1.2749		23.81	1.2904
	15.645	1.2732		15.085	1.2731		−0.04	1.2930
	19.115	1.2631		16.195	1.2734		−0.04	1.2931
	20.065	1.2633		17.295	1.2667	VI.	−0.03	1.2930
	23.22	1.2725		19.60	1.2634	Square.	+0.24	1.2916
II.	−0.155	1.2929		20.935	1.2628		4.14	1.2864
Taper.	+0.375	1.2935		22.33	1.2691		3.21	1.2893
	2.285	1.2938		23.595	1.2755		1.75	1.2911
	3.175	1.2912	IV.	−0.275	1.2923		2.18	1.2896
	3.765	1.2907	Taper.	+1.995	1.2945		4.46	1.2857
	4.65	1.2915		3.345	1.2909		6.06	1.2867
	6.125	1.2900		4.655	1.2904		7.36	1.2869
	7.39	1.2879		6.23	1.2900		8.89	1.2875
	8.22	1.2870		8.285	1.2868		10.33	1.2874
	11.315	1.2894		11.04	1.2885		10.70	1.2875
	12.525	1.2792		12.46	1.2794		13.15	1.2856
	13.97	1.2745		14.75	1.2749		14.75	1.2811
	15.445	1.2733		15.53	1.2730		16.21	1.2803
	16.585	1.2715		17.43	1.2646		17.82	1.2802
	17.74	1.2639		19.84	1.2635		19.96	1.2716
	19.255	1.2631		22.005	1.2671		20.92	1.2683
	20.51	1.2633		23.58	1.2747		21.96	1.2717
	21.605	1.2650	V.	0.14	1.2926		23.57	1.2871
	23.105	1.2726	Square.	1.16	1.2920		4.49	1.2907
	23.965	1.2756		4.18	1.2863		7.76	1.2868
III.	0.325	1.2936		5.98	1.2866		11.61	1.2873
Taper.	1.67	1.2958		7.26	1.2870		13.79	1.2832
	2.785	1.2917		10.13	1.2874		18.67	1.2789

the forms of the flaring joints at the two ends, one widening very rapidly, called *square*, the other more slowly, called *taper*.

Fig. 6 gives the complete calibration curves, reduced in scale too much to show the plotted points.

The tube was filled with mercury within the limits to which the above calibration had been extended, and the position of its ends

with regard to the reference mark was measured with the Brown & Sharpe centimeter scale and sliders (with glass windows, reference lines and mirrors) made for use with the manometers of the pressure-measuring apparatus. This thread was then forced out into a light blown glass container, and weighed with a Troemner Assay balance (sensitive to about 0.06 mg. per scale division deflection with the load used) and a set of quartz and platinum weights, No. 972, whose corrections were known by a certificate of the Normal Aichungs-Commission, dated February 18, 1898. The method of double weighing by vibrations was used, with proper precautions about dryness, handling, etc., and corrections were made for air-buoyancy. The mercury used in both long and short threads was previously cleaned a long time with mercurous sulphate and sulphuric acid; and the particular globule which composed the long thread has kept its surface untarnished after many months.

Computing was done with slide-rule or five-place logarithms, according to the degree of accuracy required. Small tables were constructed for computing the temperature corrections. Marek's values of the specific volume of mercury were used, as found in the "Smithsonian Tables." Weddle's method was used in computing the area of curves. All computations were carefully checked by repetition.

Table III. gives the data on the long thread, the lengths being each the mean of three measurements agreeing within .05 mm.

TABLE III.

CAPILLARY J_3a.

Temperature,	21.24° C.
Reference mark to end next taper seal,	23.195 cm.
" " " " " square "	23.153 cm.
Total length,	46.348 cm.
Error in length (estimated) .05 mm.,	.01 per cent.
Mass,	56.96 mg.
Error in mass (estimated) $< \frac{1}{30}$ mg.,	$<$.05 per cent.
Volume,	.004209 c.c.
Error of volume (same as error of mass),	$<$.05 per cent.

The area of curve I, between the ordinates corresponding to the ends of the long thread, was found to be 36.1558 cm^2. By the method of calibration, the volume V of the short thread was found

to be 0.00011373 cm³. The area of curve *II*, for the whole tube, was found to be 75.5127 cm². Hence, substituting in the calibration formula (11), the calibration factor

$$\frac{(v'' - v')^2}{16\pi \int_0^l \lambda^2 dl \left[\int_{l'}^{l''} \frac{dl}{\lambda}\right]^2}$$

is found to equal $3.5703 \cdot 10^{-12}$ at 21.2° C., with an estimated error of .05 per cent. To correct for expansion of the glass at $t°$, multiply by $[1 + 0.0000174\ (t - 21°.2)]$.

A uniform tube 49.6 cm. long and 0.1096 mm. in diameter would be equivalent to this one.

The average value of the meniscus height was found to be $h_1 = 0.0026$ cm., $h_2 = 0.0021$ cm.[1]

Hence the error of neglecting $(h_1^3 + h_2^3)/6a^2$ is about .01 per cent. of the average thread length; also, from curve *I* for volumes, (at the point marked *) it may be shown that $\Delta a/a$ is never greater than .00191, which justifies the neglect of both in comparison with the other terms of the formula for thread volumes (7).

After calibration the two end tubes were sealed to long pieces of the same glass, and these were bent parallel to the capillary; one also was bent back about 25 cm. from the end of the capillary, so that these connecting tubes projected side by side at the one end of the capillary, but were of nearly equal lengths. As Jena 59 III seems not to seal to soda glass such as the pressure measuring apparatus was made of, the connecting tubes of both Jena and soda glass were joined by ground joints made tight with De Khotinsky cement. This construction gave a strong air-tight joint between the two sorts of glass, and was convenient when it became necessary to take down the apparatus for reconstruction of some of its parts. The two U-shaped bends insured that all gas entering the capillary should have first been warmed to the temperature of the space surrounding the capillary and thermometer.

Electric Furnace.— The electric furnace used was made by the Cambridge Scientific Instrument Co., a platinum foil spiral on a

[1] Out of 105 measurements of meniscus heights, front and rear, the front meniscus was higher than the rear in only six cases.

porcelain tube about 61.5 cm. long and 4 cm. internal diameter, heat insulated with air space and asbestos paper sheathing; its resistance at room temperature was about 0.63 ohm; at 350°, about 1.26 ohms. It was rated to stand 110 volts and 33 amperes.

The power connections are shown in Fig. 7. *G* is a motor-generator set, giving very accurate regulation with either 55 or 110 volts at its terminals, and within 1 per cent. or 2 per cent., in spite of various heavy drafts upon the current, at the furnace terminals. Owing to the nearness of the heating coil to the inner air-bath space these variations made it necessary to have some steadying source, which was done by connecting a storage battery *B* of 10 large cells across the terminals of the furnace *F*. These cells could be arranged in any convenient combination for varying the resistance and E.M.F., and tin frames *R* in series with the generator regulated the main current till the potential difference at the furnace terminals on the average just balanced the E.M.F. of the particular combination of cells used. This steadied the furnace current considerably. The furnace was started warming some hours before a test with a current considerably larger than that needed to produce a desired temperature; when the thermometer showed that this temperature had been passed, the current was cut down and the potentials adjusted to the battery E.M.F., and the temperature allowed to settle to a steady value. As this was a very slow process toward the end, and the temperature would keep rising for a very long time, the final adjustments were made by slight variations in the resistance *R*.

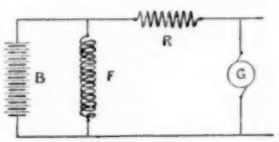

Fig. 7.

Both the thermometer tubes and the prewarming tubes for the capillary were cemented to the openings of the furnace by building about them in place plugs of asbestos stove lining; this becomes quite hard on drying, and is very resistant to the action of heat.

Platinum Thermometer and Bridge. — The temperature of the interior of the furnace, and so presumably of the gas flowing through the capillary, was measured with a resistance thermometer and Wheatstone bridge of Callendar's pattern. The galvanometer used was a Rowland D'Arsonval, read with telescope and scale, and

sensitive enough to indicate $\frac{1}{5000}$ ohm at the higest temperature observed. The resistance of the galvanometer was about 500 ohms.

Fig. 8 shows the connections. B is the dry cell, G the galvanometer, R_1, R_2 are two equal ratio coils, of No. 28 manganin wire, 0.34 mm. diameter, twisted to avoid self-induction, and shellaced; resistance about 26 ohms, and adjustment to equality within .004 per cent., by the Cary Foster method. They have been frequently tested for equality by reversal, and show no change. T is the thermometer, l_1 its leads, l_2 the compensating leads; these leads were made of twisted lampcord, equivalent to No. 12 copper wire, about 400 cm. long, carefully cut to equality in length, and when tested by interchanging on this bridge showing no difference greater than .0001 ohm. S_1, S_2 are the segments of the slide wire, of manganin, 50 cm. long, 1 mm. diameter stretched over a half meter stick graduated in millimeters; the key had a contact of manganin off the same spool as the slide wire, and its index error was determined. The slide wire was calibrated in steps by the Cary Foster method, comparing it with a standard .1 ohm coil, Hartmann & Braun No. 2115, and found to be accurately uniform within the limits of error, and to have a resistance of 1 ohm for 184.61 cm. The resistance box No. 8406 was by Leeds & Northrup, three decades, tenths, units and tens, manganin wire on wooden spools; it was guaranteed to .1 per cent. and found to be better; the error of every coil was determined to .001 ohm by the Cary Foster method, comparing the tenths decade with the above Hartmann & Braun .1 ohm coil, the units decade with O. Wolff 1 ohm coil No. 3403, the tens decade with O. Wolff 10 ohm coil No. 3204. 1, 2, 3, 4 are mercury cups, allowing of the easy reversal of the ratio coils or slide wire. With the exception of these cups and the box, battery and galvanometer binding posts all connections were soldered. The wires joining the various parts of the bridge were copper, 2.09 mm. diameter, carefully cut to equality in length.

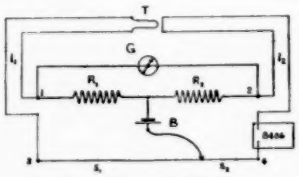

Fig. 8.

The bridge was finally tested by measuring with it the following

standard coils, and also the platinum thermometer wire calibrated by the Bureau of Standards, with the following results:

TABLE IV.

Hartmann & Braun,	.1 ohm,	No.	622,	measured	0.1006 ohms.
O. Wolff,	1 ohm,	"	3403,	"	1.000 "
Hartmann & Braun,	10 ohms,	"	675,	"	10.000− "
O. Wolff,	10 ohms,	"	3284,	"	10.000+ "
Hartmann & Braun,	100 ohms,	"	630,	"	100.008 "
Platinum wire in ice				"	5.901− "
Platinum wire in ice after heating to 570°				"	5.888− "

In use the following formula is applied, adding algebraically to the corrected box resistance $R = 0.1083x - 0.2679$, x being the scale reading, uncorrected. The formula contains all corrections. Slight variations are observed, due, no doubt, to varying plug resistances. These may amount, judging from trials at different times, to .0003 ohm, which cannot affect the measurement of temperature by more than $\frac{1}{80}$° C. Resistances measured with this bridge may be considered certain within .002 ohm.

On account of the length of the capillary it was necessary to use a means of finding the average temperature along it; as the gas was inclosed in glass walls it was decided to have the resistance wire also in a glass envelope, so as to have similar conditions.

This platinum thermometer was made of No. 35 wire procured from the Leeds and Northrup Co., of Philadelphia, as specially refined for the construction of such thermometers. A piece about 86 cm. long was fused at each end to heavy silver leads about 50 cm. long, 1.035 mm. diameter cut carefully to equality in length, and a current was passed through sufficient to raise the platinum to a white heat for several minutes. The platinum and leads were then inserted in a tube of Jena 59 III glass, which was bent at the middle into a U-shape. Another piece of silver wire, cut to a length equal to the leads, was inserted into a short tube of the same glass and the whole bent into a smaller U. The leads were secured in the tubes (during the test at the Bureau of Standards) with dental cement, later the ends of the tubes with the leads were cast in a plug of litharge and glycerine cement. Both tubes were placed in a mold of proper shape, and a stopper-shaped plug of asbestos stove-lining cast about them so that about 14 cm. of each

silver lead-wire, both actual and "dummy," would be within the furnace or temperature-bath. After drying thoroughly the apparatus was sent to the Bureau of Standards, at Washington, where it was tested according to the "S.B.P." method of Callendar and Griffiths.

Unfortunately it turned out that the annealing of the wire before this calibration was insufficient, so that after a test at about 570° the resistance in ice had changed .012 ohm. The constants for steam and ice were therefore redetermined, giving the following measurements.

Before heating to 570°.			
1. June 19, 1908, resistance in ice,	5.901	mean of	2.
After heating to 570°.			
2. June 22, resistance in ice,	5.888	mean of	4.
3. June 24, " steam at 99°.15,	8.148	"	4.
4. July 11, " in ice,	5.887	"	4.
5. Aug. 1, " in ice,	5.886	"	6.
6. Aug. 1, " steam at 99°.20,	8.148	"	5.

From the observations of the Bureau of Standards, September 17, 1907.

$$R_0 = 5.8985,\ R_{100} - R_0 = 2.2802,\ \delta = 1.50.$$

From the writer's measurements, after heating to 570°,

$R_0 = 5.888$, $R_{100} - R_0 = 2.279$, $\delta = 1.50$ (assumed unchanged from the above).

The observations on air were mostly computed with these constants.

After further heatings above 500°,

$$R_0 = 5.886,\ R_{100} - R_0 = 2.280,\ \delta = 1.50.$$

The observations on N_2O were mostly computed with these.

In the above δ is the correction constant in Callendar's equation

$$t = \frac{R_t - R_0}{\frac{R_{100} - R_0}{100}} + \delta \cdot \frac{t}{100} \cdot \left(\frac{t}{100} - 1 \right).$$

The unusual method of connection, with the galvanometer in circuit all the time, the battery connected by a key to the slide-wire, is justifiable only in case self-induction is negligible. That this was so was determined by inserting in place of the platinum

thermometer wire a similar loop of copper wire, of too low resistance to be warmed by the measuring current, and finding that all evidences of self-induction were absent, it was concluded that a small "kick" given by the galvanometer on pressing the battery key, and which was always in the same direction, but only evident when the bridge was nearly in balance, must be due to this cause: the platinum is warmed by the measuring current, and so when the battery key is first pressed the resistance is lower than it is a little later. Hence the galvanometer first throws slightly in the direction indicating low resistance, then rapidly drifts the opposite way. This behavior was invariable, the directions being reversed on reversing the battery.

The arrangement is very convenient, once it is shown by test to be allowable, as the thermal E.M.F.s of the system produce a constant deflection which serves as a zero. The balance was assumed to be found when, on pressing the battery key, no perceptible kick was observed, but after hesitating a second or two the galvanometer would drift toward the high temperature direction. At the definite and constant temperatures of steam or melting ice results were thus obtained consistent to within the limits of the bridge calibration.

Method of Filling with Air or Gas. — To fill the apparatus with air or gas, open *6* and lower *G* to give free connection between *B* and *A*. Turn *1* to connect t_1 to pump through *3*, and at the same time t_2 through *2* to the purifier. Lower the buckets to empty the pipettes. Close *6*. Exhaust to as high a vacuum as the pump will give; 0.55 cm. was found attainable with five minutes pumping. Admit the air or gas by opening the cock of the purifier. Exhaust and fill twice more to wash out impurities. Close *2*; set *1* across, to put t_1 and t_2 in communication. Raise buckets, giving excess internal pressure, which may be reduced by opening *3* and pumping out. Open *6*, and raise *G*, so that the mercury stands in *A* and *B* at the desired level; close *6*.

III. Preliminary Tests on Air and Nitrous Oxide.

Purification of Air. — The apparatus for purifying air consisted of three Drechsel washing flasks and two stopperless blown glass calcium chloride tubes, with stopcocks on each side of the calcium

chloride tubes. The ground stoppers of the Drechsel flasks were tightened, the first with stopcock grease, the second and third with shellac. The second and third Drechsel flasks and the calcium chloride tubes were connected with blown joints. The first two Drechsel flasks contained a strong solution of sodium hydrate, the third concentrated sulphuric acid. Air was allowed to bubble slowly through the series of reagents, until the flow ceased. It was assumed that removal of water vapor and carbon dioxide was complete without further tests.

Preparation and Purification of Nitrous Oxide. — Fig. 9 shows the

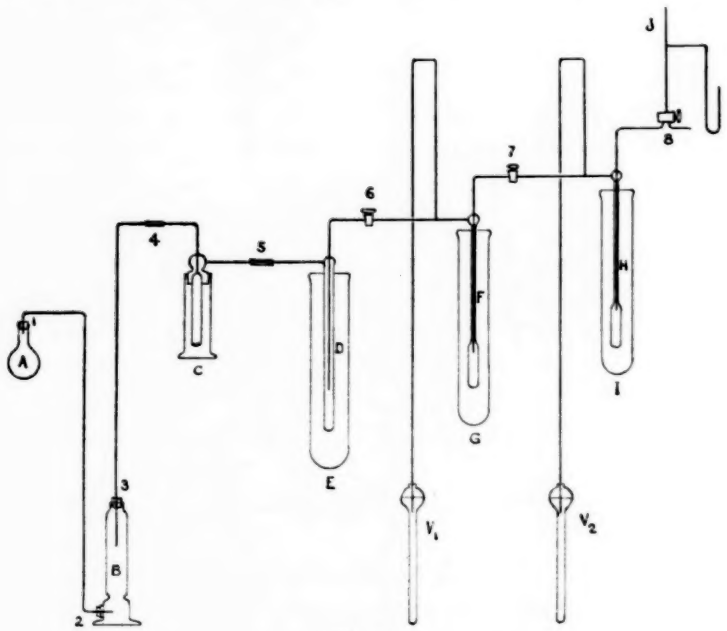

Fig. 9.

apparatus constructed for this purpose. The general plan of the method is that of Moissan, with slight modifications to increase its safety and convenience.

A is the generating tube, containing commercial ammonium nitrate (Merck's), *B* is a drying jar, containing calcium chloride, *C* a Drechsel bottle, with concentrated sulphuric acid. *D*, *F* and *H* are congealing tubes through which the gas must pass in tandem; they

[1] H. Moissan, Comptes Rendus, 137, p. 363.

are surrounded by Dewar flasks *E*, *G* and *I*, which can be set at any desired heights or removed from the congealing tubes altogether. *E* contains a paste of carbon dioxide snow and alcohol, *G* and *I* contain liquid air. *1* is a rubber stopper, tied down, *2* a rubber and *3* a cork stopper, both well shellacked, *4* and *5* are rubber connections; beyond these there are only glass stopcocks *6*, *7* and *8* and solid blown glass connections, clear to the the air pump and viscosity apparatus. Between *6* and *F* and between *7* and *H* rise up T-tubes which bend over and descend with vertical ends over a meter long, dipping below into a mercury valve tubes V_1 and V_2. These are adjustable for height, so that sufficient internal pressure may be secured up to 25 cm., and they allow of complete exhaustion with no danger of leakage. *J* is the lead tube connecting with the air pump and experimental apparatus.

Gentle warming of the ammonium nitrate in *A* melts it and sends off a slow current of mixed nitrous oxide and water vapor. The valves are set so that gas passes out at V_2 more easily than at V_1. Water vapor is removed in *B*, *C* and *D*, and the nitrous oxide is frozen in *F*. Cock *6* is then closed, *7* and *8* are opened, and the apparatus is exhausted. By lowering *G* to a proper point the nitrous oxide gradually sublimes over into *I*, where it is again frozen. Cock *7* is then closed, and the exhaustion repeated, after which lowering *I* allows the N_2O to sublime over and fill the viscosity apparatus. As the pressure rises the solid N_2O suddenly melts; when the pressure is sufficiently high it passes off through the valve V_2.

In this preliminary work no analysis of the air or gas used was made, time and facilities both being lacking. In future work this will not be neglected; though it should be remarked that Moissan, in his note on the method, says that with a proper choice of refrigerants all perceptible impurities were removed from his samples, and this with little reference to the method of generation, which might be such as to give very impure products.

Summaries of Results on Air and Nitrous Oxide. — The two following tables give values obtained in trial runs with air and nitrous oxide. While the work was carefully done, and extends to temperatures 200° higher than any which have ever before been attempted with glass capillaries, or indeed with any calibrated capillaries,

it is to be regarded as only preliminary for several reasons. The most important of these is the regulation of the temperature of the pipettes, which is not yet sufficiently under control to insure without fail an accuracy of much better than a half per cent. Even at high temperatures the temperature of the furnace generally was under sufficiently good control to give an accuracy of one tenth per cent.

TABLE V.

Viscosity of Air.

Run No.	Temp. °C.	$\eta \cdot 10^4$	θ	$\frac{\theta^{\frac{3}{2}}}{\eta} \cdot 10^{-7}$
1	24.8	1.845	297.8	2.788
2	24.0	1.835	297.0	2.790
3	481.7	3.519	754.7	5.890
4	479.3	3.222	752.3	6.415
5	22.6	1.848	300.0	2.814
6	23.7	1.845	296.7	2.774
7	347.0	3.008	620.0	5.13
8	25.3	1.915	298.3	2.920
9	25.7	1.829	298.7	2.825
10	489.4	3.562	762.4	5.915
11	496.9	3.607	769.9	5.83
12	498.0	3.551	771.0	6.03
13	501.2	3.606	774.2	5.97
14	23.0	1.849	296.0	2.755
15	220.2	2.735	493.2	4.01

Table V. contains the results on air, which are also graphically displayed in Fig. 10, curve *I*. The points on this curve have standing by them numbers indicating how many runs are averaged to get the coördinates of the points.

TABLE VI.

Viscosity of N_2O.

Run No.	Temp. °C.	$\eta \cdot 10^4$	θ	$\frac{\theta^{\frac{3}{2}}}{\eta} \cdot 10^{-7}$
1	26.2	1.517	299.2	3.606
2	75.8	1.739	348.8	3.746
3	141.6	1.970	414.6	4.285
4	183.1	2.161	456.1	4.51
5	25.0	1.498	298.0	3.438
6	224.4	2.348	497.4	4.73
7	289.9	2.610	562.9	5.12
8	413.6	3.073	686.6	5.86

Table VI. contains the results on nitrous oxide, and curve *II* of Fig. 10 shows the same. In the plotting of the points of this curve the points marked with a ring are those of the present research, those with a cross are von Obermayer's. Points at or below the critical temperature are neglected.

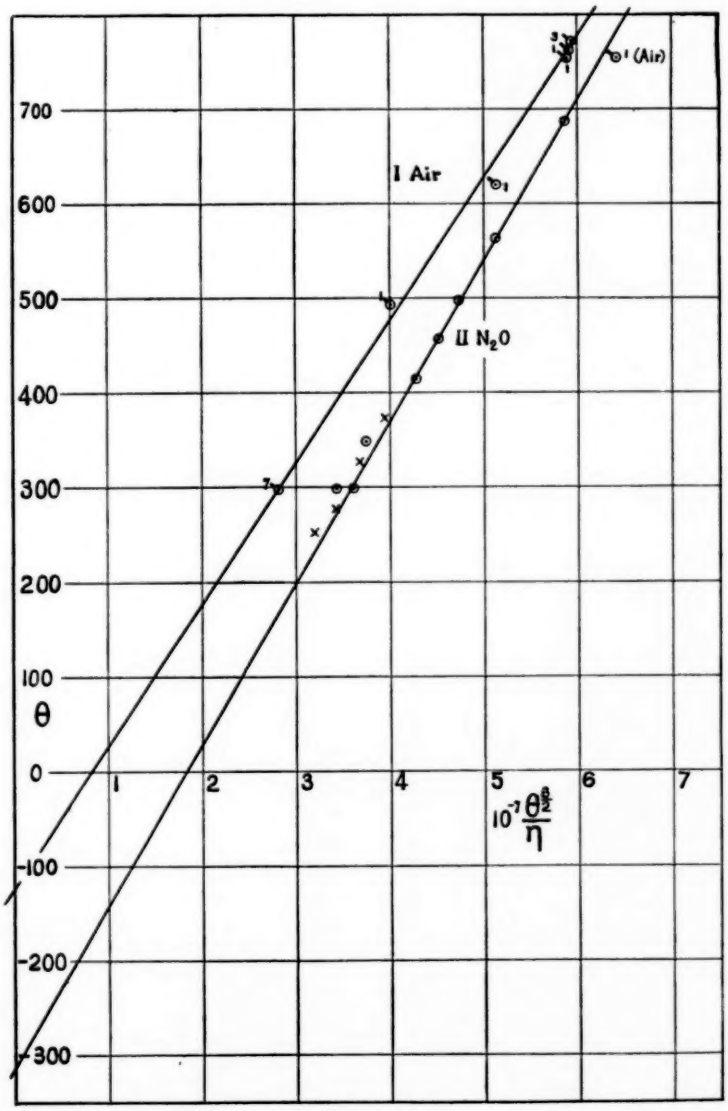

Fig. 10.

In computing these data the values of temperature and viscosity have been found using five-place logarithms; those of $\theta^{\frac{3}{2}}/\eta$ with a slide-rule. The values of η are not corrected for slip. The curves are drawn by estimation.

From the curves we derive the following constants:

Air	$C = 124$	$K = 150.2 \cdot 10^{-7}$	W. J. F.
N_2O	$C = 314$	$K = 170.7 \cdot 10^{-7}$	W. J. F.
	$= 312$	$= 173.8 \cdot 10^{-7}$	von Obermayer.[1]

It is hoped to continue the work with certain improvements in the apparatus and greater accuracy in the corrections, and to determine viscosity coefficients for all the gases to which the apparatus is adapted.

PHYSICAL LABORATORY,
CORNELL UNIVERSITY.

[1] See Part I., p. 393.

THE EFFECT OF TENSION ON THERMAL AND ELECTRICAL CONDUCTIVITY.

By N. F. Smith.

THE development of the electron theory of metallic conduction has given new interest to measurements of thermal and electrical conductivity. Since it has been shown that the ratio of the two conductivities is approximately constant for all metals it is natural to expect that anything which produces a change in the one would cause a corresponding change in the other. The effect of tension on the electrical conductivity of wires has been measured by Cantone, Williams,[1] Ercolini,[2] Gray[3] and others. These measurements have been made on small wires stretched by correspondingly small weights. It has been shown that, in general, the resistance of such wires increases when under tension; that up to the elastic limit, the increase in resistance is nearly proportional to the stretching force; that beyond this state, the change is roughly proportional to the elongation. Bismuth shows a change in the opposite direction. So far as the writer knows, no measurements have been made upon the corresponding thermal problem. In 1888 F. Kohlrausch[4] compared the thermal and electrical conductivities of various specimens of steel when tempered to different degrees of hardness. He found that while the conductivities changed by as much as 50 per cent., their ratio remained practically unchanged.

The present investigation was undertaken to determine what changes, if any, are produced in the thermal conductivity of metal bars when they are subjected to a stretching force, and to compare these changes with those which are produced in their electrical conductivity. Two distinct problems were therefore presented, the one involving thermal measurements, the other, electrical. In neither

[1] W. Ellis Williams, Phil. Mag., 13, p. 635, 1907.
[2] G. Ercolini, N. Cimento, 14, p. 537, 1907.
[3] T. Gray, Trans. Roy. Soc. Edin., 1880.
[4] F. Kohlrausch, Wied. Ann., 33, p. 678, 1888.

case was any attempt made to determine the absolute value of the conductivity, but merely the per cent. of change produced by the stretching force.

THE METHOD.

The method adopted for the thermal problem was substantially that of Wiedemann and Franz. Two bars A and B, of the same size and material and as nearly alike as possible in every respect, were heated at one end to a constant temperature. When the steady state was reached, a point was determined on B whose temperature was the same as that of a fixed point on A. B was then subjected to a stretching force while the condition of A remained unchanged. When the steady state was again reached, a new point was determined on B having the same temperature as the fixed point on A. Assuming that the conductivity is proportional to the square of the length from the hot end to this point of constant temperature, we have a means of measuring the change produced by the force applied. While this assumption is not rigorously true, yet, since the changes in conductivity are relatively small, the error introduced by its adoption is less than the experimental errors.

It seemed desirable to study the changes in the electrical conductivity of the same bars used in the thermal problem and under the same conditions. Accordingly the electrical resistance of a measured length of the bar B was compared with that of approximately the same length of bar A as each successive tension was applied to B. The method here used was a modification of the Kelvin "double-bridge method" due to J. H. Reeves.

THE APPARATUS.

After many modifications the apparatus finally used was constructed as shown in plan in Fig. 1, and in elevation in Fig. 2. Round bars were employed $\frac{5}{16}$ inch in diameter and about $4\frac{1}{2}$ feet long. The bars A and B were mounted horizontally side by side about 3 inches apart. At the left-hand end they passed completely through a cast-iron box C, each edge of which was 8 inches and whose sides were 1 inch thick. This rested on a tripod stand over a Bunsen burner. The box was provided with a cast-iron lid and was thickly wrapped in asbestos. On the inside of the box the bars

were clamped between two heavy pieces of copper *D*, one above the other, each 5 inches long, $1\frac{1}{2}$ inches wide and 1 inch thick. These were bored out to receive the bars and held together by bolts. The bars were carefully insulated from the box and from the copper clutch by thin sheets of mica. A hole in the center of the copper

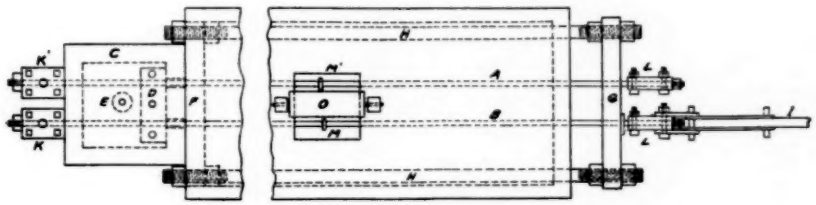

Fig. 1.

clutch received the bulb of a thermometer whose stem projected through the cover of the box. The copper clutch was maintained at a constant temperature by a thermostat. This consisted of a mercury bulb *E*, placed at the bottom of the box directly over the flame, having a capillary stem projecting above the cover. A wire inserted in the top of the stem touched the mercury when a certain

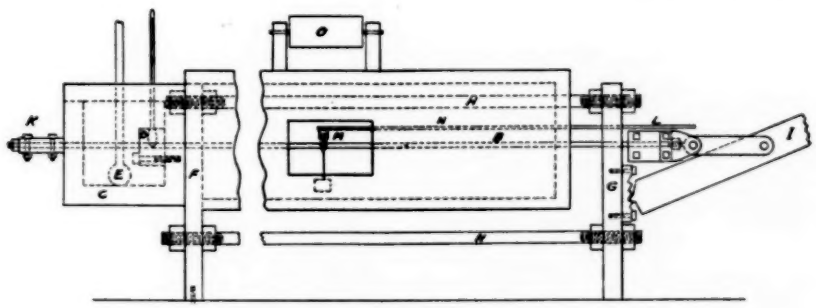

Fig. 2.

temperature was reached. This closed an electric circuit and shut off the gas from the burner. As soon as the temperature commenced to fall the circuit was opened and the gas turned on and lighted by a small pilot light which burned continuously. Some difficulty was experienced in making the mercury bulb respond quickly enough to the changes of temperature. It was at first submerged in a bath of paraffin, but this was given up because of the

smoke given off when somewhat high temperatures were employed. A bath of mercury was next used, but this partly vaporized and then condensed in the cooler parts of the box, destroying the insulation of the bars. Finally, the alloy used in making fuse wire was employed. This melted at the temperatures used, but did not vaporize and secured intimate thermal connection between the bulb and the bottom of the iron box. With this arrangement the thermometer, graduated to degrees, showed no measurable variation. If there was a slight variation in the temperature of the copper clutch it affected both bars alike and hence introduced at most a very slight error in the results.

In order to apply tensions equal to the breaking strength of the bars it was necessary that they be mounted in an extremely rigid frame. This was constructed as follows: A cast-iron plate *F*, 11 inches wide, $1\frac{3}{4}$ inches thick and 13 inches high, was mounted upright on a table immediately in front of the iron box. A similar plate *G* was mounted parallel to the first and about 36 inches in front of it. The bars under observation passed through suitable holes in these plates, being insulated from them by fiber tubing. Four $\frac{3}{4}$-inch iron rods *H*, connected the two plates, and double nuts at each end of each rod held the plates securely in position. Two $\frac{3}{8}$-inch iron rods were screwed into the bottom of the plate *F* and made fast to the floor joists below. Two other rods $\frac{1}{2}$ inch in diameter passed through *F* horizontally, then through a 12-inch brick wall of the building, and were made fast to a large iron plate on the opposite side.

The stretching force was applied by means of a lever *I*. This consisted of a bar of the best tool steel, 4 feet long, $\frac{3}{4}$ inch thick and $2\frac{1}{4}$ inches wide. A steel knife-edge $2\frac{1}{2}$ inches long was attached at right angles to the lever at its lower end. To the plate *G* was screwed another iron plate *J*, insulated from *G* by sheets of mica. The plate *J* had several horizontal notches cut in it, in which the knife edge on the lever could rest, serving as a fulcrum.

To hold the bar *B* from slipping and to afford a means of attachment to the lever, a friction clutch *K*, *L* was provided for each end. Each of these was constructed from two pieces of iron $\frac{1}{2}$ inch thick, 2 inches wide and 3 inches long. These two iron blocks,

with a thin sheet of metal between them, were first securely bolted together by four $\frac{3}{8}$-inch steel bolts. A $\frac{5}{16}$-inch hole was then drilled lengthwise through the center. The sheet metal was removed and the bars ground with emery between the parts of the clutch. When tightly bolted together these clutches would not slip even when a tension was applied sufficient to break rods of steel. However, as a further precaution threads were cut for a half inch at each end of the bar and steel nuts screwed on outside the friction clutches. The clutch L was connected to the lever I by a link made of two pieces of strap iron 1 inch wide and $\frac{3}{16}$ inch thick, one piece being placed on each side of the lever. Steel pins connected this link with the clutch and the lever. By means of the nuts on the rods H, the plate G could be moved parallel to itself and so adjusted that the lever always exerted a horizontal pull upon the bar. A box for carrying weights hung from the end of the lever and could be gradually lowered or raised by means of a jack-screw.

In order that the bar A might be under conditions as nearly as possible identical with those of B, its two ends were provided with a similar pair of friction clutches and lock-nuts. This was done in order that any changes in room temperature might produce the same effect upon the outer ends of the two bars.

To protect the bars from air currents, they were enclosed by a tight wooden box 12 inches wide, 9 inches high and 34 inches long, extending from the iron plate F to within two or three inches of plate G. The top of this box was hinged and could be lifted to afford access to the bars and glass windows in the top and side afforded means of observation.

Observations of temperature were made by means of thermo-electric couples. According to the method employed, it was not necessary to make measurements of temperature upon either bar, but merely to determine two points, one on each bar, which had the same temperature. Two thermo-electric couples made of No. 40 iron and nickel wire were mounted as shown in Fig. 3. M and M' are pieces of hard rubber from which are hung equal weights, W and W'. These could slide along on the tops of the bars A and B. A piece of nickel wire connected M and M', passing through a small

hole in each and ending in a loop. A piece of iron wire was similarly mounted on each side, the junction of iron and nickel coming directly on top of the bar. The iron wires passed out through holes in the bottom of the wooden box and were soldered to copper wires which ran to the galvanometer. The two junctions of iron and copper were immersed in a bottle nearly filled with kerosene. To avoid the possibility of thermo-electromotive forces being introduced at the connection with the galvanometer, the instrument was enclosed in a wooden box and the mirror observed through a small opening in the front. All parts of this enclosure were doubtless at substantially the same temperature. Before each observation, the bottle of kerosene was thoroughly shaken to insure equality of temperature. It is therefore evident that if the two iron-nickel junctions are at the same temperature, the galvanometer will show no deflection. The couple on the bar *B* could be moved along in either direction by means of two prongs projecting at right angles to a light brass rod *N* which passed through the end of the box and the iron plate *G*. The galvanometer used was a four-coil Kelvin having the coils in parallel so that its resistance was about 6 or 7 ohms. The sensitiveness was such that with the two couples at about 50 cm. from the copper clutch settings could be made to within about .2 mm.

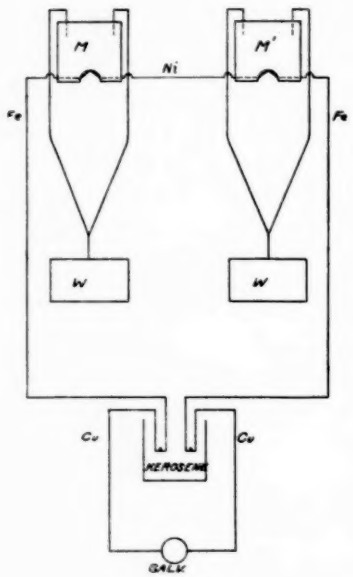

Fig. 3.

To observe the relative position of the two couples, a white paper index was pasted to the top of each piece of hard rubber. A mirror *O* was mounted on a horizontal axis parallel to the bars above the window in the top of the wooden box. This served to throw the image of either index into the field of the telescope of a cathetometer mounted horizontally parallel to the bars. The windows in the box were kept covered except when readings with the cathetometer were being made.

To measure the elongation of the bar under tension, a telescope was mounted which could be moved horizontally by a micrometer screw reading to hundredths of a millimeter. This was focused simultaneously upon a scratch on the bar B and a brass index securely clamped to A. Since the whole frame yielded somewhat under stress the elongation was determined from the change in the distance between these two index marks. The probable error of setting on each mark was about .01 mm.

Heavy copper lead wires were soldered to the friction clutches L and L', while holes drilled in the tops of K and K' served as mercury cups through which the two bars A and B could be electrically connected by a piece of copper rod bent twice at right angles.

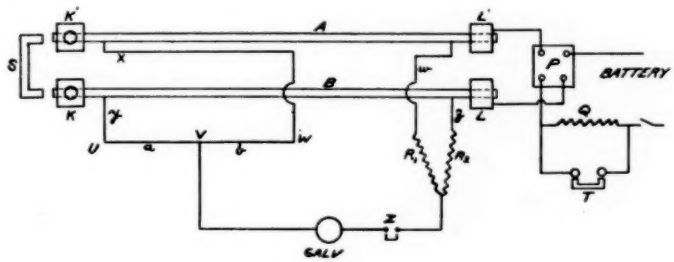

Fig. 4.

The arrangement of apparatus for the comparison of resistances is shown diagrammatically in Fig. 4. A number of storage cells could be connected with the two bars A and B through the commutator P. A resistance Q (consisting of two incandescent lamps in parallel) was introduced into the circuit. This resistance could be completely cut out by inserting the copper connector T in its mercury cups. After trying various forms of movable contacts, the fine lead wires x, y, z, w were finally soldered directly to the bars. Great difficulty was experienced in eliminating the effects of thermo-electromotive forces developed at various contacts of dissimilar metals. For this reason the lead wires were wherever possible made of the same material as the bars to which they were attached. The other ends of x and y were joined to the terminals of a slide-wire bridge whose resistance wire a, b was wound on a revolving cylinder and the whole enclosed in a wooden box so that the three

junctions U, V, W should be at the same temperature. The free ends of z and w were joined to the terminals of two standard resistance boxes R_1 and R_2. R_1 was keep fixed at 1,000 or 2,000 ohms while R_2 was adjusted. Since it was found that small thermo-electromotive forces were developed by the pressure of an ordinary contact key, a key was constructed at Z consisting simply of a bent copper wire mounted in a piece of fiber on a spring and bridging the gap between two mercury cups. This insured always a certain and uniform contact and one free from any E.M.F. Adjustments of the sliding contact V and of the resistance R_2 were made until the galvanometer showed no deflection whether the connector S was in the mercury cups or removed. When S was removed T was also removed and a relatively small current employed. When S was inserted T was also inserted and a current of from 10 to 20 amperes sent through the bars. In this way the resistance of the portion of B between y and z was compared with that of A between x and w. The direction of the current was then reversed by the commutator P and the measurements repeated. The results obtained by reversing the current seldom differed by more than one part in two thousand and in many cases no measurable difference could be observed.

THE OBSERVATIONS.

The procedure in making a set of observations was as follows: An electro-magnet connected with the program clock served to turn on the gas at any hour desired. Since four or five hours was required before the steady state was reached, the mechanism was usually set for three o'clock in the morning. The thermostat was adjusted to maintain the temperature desired in the experiment by setting the contact wire at the proper height in the tube. At eight or nine o'clock in the morning observations were usually begun. The thermo-electric couple on B was adjusted till the galvanometer showed no deflection and the position of the two couples read by the cathetometer. The relative position of the index marks on the two bars was read by the micrometer telescope. The electrical resistances of fixed lengths of the two bars were then compared. A suitable weight was then hung from the end of the lever and the apparatus left for at least an hour, or until observation

showed that the steady state was again reached. The series of observations was then repeated. This was continued till the maximum tension which the bar could safely stand was reached when the weights were gradually removed and the set of observations made with decreasing tension.

Results.

Measurements have been made on bars of iron, steel, copper, brass and a few on aluminum and zinc. One typical set of measurements is given in full in Table I. In this and subsequent tables T

Table I.

Iron Bar B_{11}. *July* 10, 1908. *Temperature* 200°.

T	Micrometer Readings.		ΔL	Resistance.		
Kg.	A	B	$=B-A$	Direct.	Reversed.	Ave.
	mm.	mm.	mm.			
0	2.50	2.50	0	1999.0	1999.0	1999.0
40	3.95	4.90	.95	2005.0	2005.2	2005.1
80	4.91	7.34	2.43	2017.0	2018.0	2017.5
40	4.45	6.54	2.09	2012.0	2013.0	2012.5
0	3.33	4.63	1.30	2003.0	2003.4	2003.2

Couples.			l	l^2	K	Δs	σ
A	B	$B-A$	$=50$ cm. $-[B-A]$				
cm.	cm.	cm.					
3.19	3.54	.35	49.65	2465	1.000	0	1.0000
3.04	2.96	−.08	50.08	2508	1.017	.000235	1.0020
2.99	2.53	−.46	50.46	2546	1.033	.000601	1.0065
3.03	2.97	−.06	50.06	2506	1.016	.000541	1.0044
3.03	3.35	.32	49.68	2468	1.001	.000321	1.0070

Length of bar between points from which ΔL was measured, 111.5 cm.
Length of bar between points from which resistance was measured, 85.4 cm.
Value of Poisson's ratio employed in computing Δs, .277.

is the load in kilograms hung from the lever. Where the load is marked 0, the bars were stretched by a force due to the weight of the lever. This was equivalent to a load of about 5 kg. To express the tension in kilograms per square centimeter, T must be multiplied by 29.3 ΔL is the elongation in centimeters; l, the distance from the edge of the copper clutch to the position of the thermo-electric couple on bar B; K, the ratio of the thermal con-

ductivity of the bar to its conductivity before any tension was applied; σ, the corresponding ratio of specific resistances; $\triangle s$, the relative change in cross-section of the bar.

TABLE II.

Further Measurements on B_{11}.

T	ΔL		σ	T	ΔL	K	σ
July 11, Temp. 195°.				July 14, Temp. 196°.			
Kg.	mm.			Kg.	mm.		
0	1.14	1.002	1.0009	0	1.68	.9952	1.0019
40	2.00	1.018	1.0019	40	2.32	1.019	1.0030
80	2.85	1.022	1.0053	80	2.90	1.024	1.0047
40	2.40	1.007	1.0042	40	2.35	1.019	1.0031
0	1.61	1.000	1.0019	0	1.70	1.003	1.0017
July 13, Temp. 196.5.				July 15, Temp. 196°.			
0	1.62	1.006	1.0019	0	1.70	1.003	1.0018
40	2.42	1.016	1.0042	40	2.36	1.022	1.0037
80	2.88	1.019	1.0044	80	2.95	1.030	1.0047
40	2.31	1.010	1.0031	40	2.35	1.022	1.0035
0	1.76	.9983	1.0020	0	1.79	.9972	1.0016

In computing the relative thermal conductivity no correction has been made for the decrease in cross-section of the bar as it is stretched. This change in cross-section may be computed from the value of Poisson's ratio for the given material and the correction applied. It is found, however, that it affects only the fourth decimal figure and hence is beyond the limits of accuracy of the experiment. Since the value of σ is computed to four decimal figures, the correction is there computed and applied.

In Table II. are given in condensed form the results of four other sets of measurements on the same iron bar B_{11}, made after those given in Table I. The results for other bars of different metals are given in the other tables. Some of the results are plotted in the accompanying curves.

Many other measurements were made on other bars of the same metals, a total of eighteen different bars being employed. The results obtained for the other bars were substantially the same as those recorded. The difference in the behavior of different bars of the same metal was no greater than the difference in successive

TABLE III.

Steel Bar, B_{10}.

T	ΔL	K	σ	T	ΔL	K	σ
June 25, Temp. 201°.				June 30, Temp. 205°.			
0	0	1.000	1.0000	0	.59	1.051	1.0018
40	.43	1.034	1.0009	40	1.30	1.070	1.0018
80	.88	1.050	1.0008	80	1.79	1.079	1.0021
120		1.047		120	2.19	1.080	1.0022
Weigh hung on over night.				175	2.85	1.076	1.0039
June 26, Temp. 204°.				July 1, wt. hung on over night. Temp. 203°.			
175	2.42	1.050	1.0017	175	2.91	1.076	1.0028
120	1.87	1.059	1.0023	120	2.11	1.064	1.0023
80	1.35	1.057	1.0021	80	1.42	1.068	1.0018
40	.74	1.056	1.0014	40		1.058	
0	.33	1.041	1.0012	0	.59	1.042	1.0009
June 27, Temp. 204°.				July 2, Temp. 202°.			
0	.28	1.039	1.0004	0	.59	1.041	1.0014
40	1.22	1.057	1.0010	40	1.31	1.063	1.0002
80	1.69	1.071	1.0014	80	1.68	1.073	1.0012
120	2.16	1.072	1.0020	120	2.06	1.078	1.0017
175	2.69	1.069	1.0058	175	2.76	1.080	1.0028
une 29, weight hung on from Saturday until Monday. Temp. 201°.				July 3, weight hung on over night. Temp. 202°.			
175	2.75	1.070	1.0056	175	2.76	1.085	1.0037
120	2.01	1.062	1.0053	120	2.07	1.079	1.0034
80	1.24	1.057	1.0050	80	1.43	1.071	1.0035
40	.91	1.067	1.0044	40	1.04	1.070	1.0022
0	.59	1.052	1.0027	0	.59	1.049	1.0013

measurements on the same bar. The experiment has been set up twice with different apparatus and in all cases has yielded results of the same order of magnitude.

CONCLUSIONS.

It is probable that the apparent change in the thermal conductivity of the same bar under the same tension from day to day is due in part to changes in the character of the surface of the bar. No attempt was made to prevent oxidation, which affected somewhat the surfaces of both the bars compared. If the radiating power of one surface changed more than that of the other an apparent change in conductivity would result. It is hardly probable that this change

TABLE IV.

Copper Bar, B_{14}.

T	ΔL	K	σ
July 27, Temp. 152°.			
0	0	1.000	1.0000
30	1.63	1.026	1.0001
55	3.40	1.030	1.0010
30	2.66	1.028	.9993
0	1.74	1.020	.9985
July 28, Temp. 152°.			
0	1.67	1.024	.9985
30	2.52	1.036	1.0000
55	3.67	1.030	1.0016
30	2.97	1.037	.9999
0	1.90	1.023	.9982
July 31, Temp. 150°.			
0	1.92	1.021	Resistance not measured.
30	2.75	1.032	
55	3.14	1.024	
30	2.52	1.028	
0	1.83	1.018	
30	2.68	1.029	
55	3.52	1.020	
30	2.66	1.021	
0	1.86	1.010	

T	ΔL	K	σ
August 4, Temp. 150°.			
0	1.83	1.015	Resistance not measured.
20	2.51	1.019	
30	2.74		
40	3.08	1.028	
55	3.40	1.016	
40	2.98	1.026	
30	2.76	1.018	
0	1.98	1.013	
August 5, Temp. 151°.			
0	1.96	1.010	Resistance not measured.
20	2.45	1.018	
30	2.60	1.015	
40	2.96	1.014	
55	3.65	1.007	
40	3.02	1.018	
30	2.97	1.010	
0	2.05	1.010	

during the progress of a single experiment was sufficient to affect the comparison of these results among themselves. An inspection of the tables and the curves shows that in every case the thermal conductivity of the bars increases when a moderate tension is applied. As the limit of elasticity is reached this increase approaches a maximum. As the tension exceeds this limit, the conductivity remains practically constant in the case of the more elastic metals, such as steel and brass, or begins to diminish in the case of the softer metals. This is shown in some of the tests on iron bars and particularly in the case of copper. After a bar has been stretched its conductivity does not immediately return to its former value. This is most noticeable in steel and brass. In the softer metals the return to the original value is more perfect. It is probable that steel and brass would regain their original conductivity if given

TABLE V.

Brass Bar, B_{18}.

T	ΔL	K	σ	T	ΔL	K	σ
August 12, Temp. 156°.				August 13, Temp. 154°.			
	mm.				mm.		
0	0	1.000	1.0000	60	2.50	1.041	1.0099
30	1.46	1.027	1.0000	30	1.53	1.037	1.0077
60	2.19	1.030	1.0023	0	.83	1.024	1.0080
90	3.21	1.033	1.0047	August 14, Temp. 154°.			
60	2.78	1.035	1.0012		mm.		
30	1.94	1.025		0	.74	1.020	1.0082
0	.92	1.015	1.0000	30	1.71	1.039	1.0094
August 13, Temp. 154°.				60	.50	1.041	1.0100
				90	3.19	1.038	1.0106
0	.81	1.021	1.0086	60	2.38	1.036	1.0096
30	1.88	1.037	1.0088	30	1.54	1.034	1.0097
60	2.60	1.041	1.0099	0	.71	1.018	1.0092
90	3.22	1.042	1.0102				

sufficient time, but this has not been tested by experiment. The total change in the conductivity of steel may amount to 7 or 8 per cent.; that of iron to 4 or 5 per cent.; brass, about 4 per cent. and copper 2 or 3 per cent. A few measurements were made on aluminum before the apparatus was put in its final form. These showed an increase in thermal conductivity of about one half per cent. under the maximum tension. Measurements were attempted upon bars of zinc but these proved unsatisfactory. The bars were not of uniform structure or cross-section, and stretched continuously and slowly under any weight applied, not being able to support even the weight of the lever. At the same time they showed apparently a larger increase in conductivity than bars of any other material tested. Since constant conditions were never attained, no great confidence can be placed in the results.

In every case the electrical resistance increased with increasing tension ; that is, the conductivity diminished. These results are in substantial agreement with those obtained by other experimenters on the resistance of wires. Of course the conditions of this experiment were not suited to a highly accurate determination of this effect. It is, however, of interest to note that the changes in the thermal conductivity produced by moderate tensions are opposite in direction

to those in the electrical and are of an order of magnitude about ten times greater. Some rather rough experiments upon the effect of

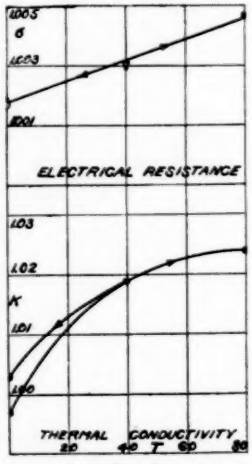

Fig. 5. Iron Bar B_{11} (July 14).

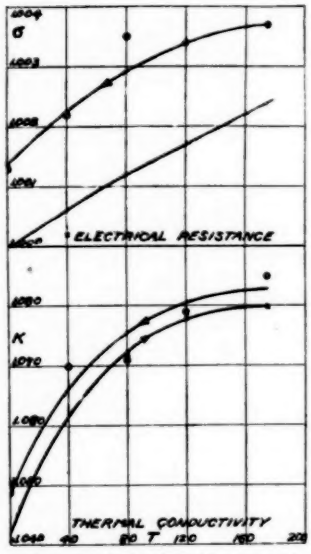

Fig. 6. Steel Bar B_{10} (July 2 and 3).

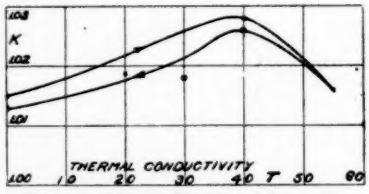

Fig. 7. Copper Bar B_{14} (Aug. 4).

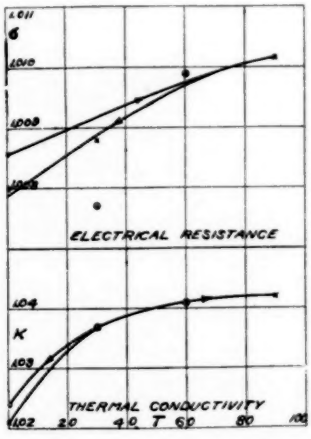

Fig. 8. Brass Bar B_{18} (Aug. 13).

bending bars of iron and copper so that a large permanent deformation was produced seemed to show a marked decrease in thermal conductivity of from five to eight per cent. This is to be expected

from the behavior of copper bars when stretched beyond their elastic limit. It is hoped at some future time to make a further study of this effect and also to investigate the changes produced by torsion.

My thanks are due to Mr. Walter Kachelski, the college mechanician, for advice and assistance in the construction of the apparatus.

OLIVET COLLEGE,
August 21, 1908.

A STUDY OF OVERCAST SKIES.

BY EDWARD L. NICHOLS.

IN two recent communications I have described the results of certain measurements of the visible spectrum of the light from the sky[1] and have made comparisons between the spectrum of daylight and that of various artificial sources of illumination.[2] In the present paper, the spectrum of the light of overcast skies, is more particularly considered in its relations to the spectra obtained from the cloudless sky and from skies in intermediate stages.

As in the measurements already described the instrument used was a spectrophotometer of the Lummer-Brodhun type so arranged that one collimator pointed to the zenith while the other, which was horizontal, received the light from a comparison source of nearly constant intensity and composition. This comparison source was an acetylene flame. Measurements were made throughout the visible spectrum from the extreme red at .74 μ to the extreme violet at .38 μ thus including in the observations two regions lying close to the boundaries of the spectrum which had hitherto been comparatively neglected. The results obtained, as in the previous papers just referred to, are presented in the form of curves in which abscissæ are wave-lengths and ordinates give the brightness of the spectrum of skylight in terms of the brightness of the corresponding region in the spectrum of the acetylene flame. The scale adopted is entirely arbitrary. The brightness of the comparison spectrum was adjusted to a convenient intensity by the interposition of a diaphragm in front of the flame and of a milk glass screen of neutral tint, the nonselective character of the transmitting power of which had been ascertained by careful observation. Although many studies of the quality of the light from clear skies have been made, generally for the purpose of testing Rayleigh's theory but little attention has been given to the light of clouded skies. So far as I am aware in-

[1] Nichols, PHYS. REV., Vol. XXVI., p. 497.

[2] Nichols, Transactions of the Illuminating Engineering Society, Vol. III., p. 301.

deed the only definite spectrophotometric data are those published by Crova in the course of his extended and systematic observations on the skies at Montpellier in France.[1]

The results to be described in this paper were made during a vacation journey in Europe in 1907.

Measurements of the spectrum of the light from the zenith taken at times when the sky was completely overcast gave curves notable for their simplicity and for their similarity to one another. The type of these curves is sufficiently represented in Fig. 1 where the curve V is for an overcast sky studied in Vienna in June, 1907, and Z represents the character of the light obtained from a similar sky

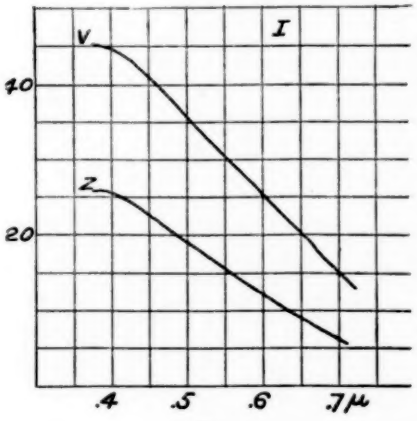

Fig. 1.

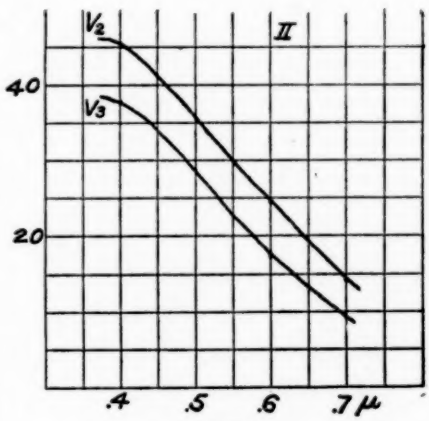

Fig. 2.

measured at Zell am See, Austria, in July of that year. The former curve was taken just before noon on a rather bright but completely cloudy day, the latter about 6 P. M. when the sky was heavily overcast and threatening rain. While these two skies differ in brightness approximately in the ratio of one to two they indicate remarkable similarity as to the composition of the light coming from the clouds.

It was noted by Crova in the course of his investigation that overcast skies differ but little in composition from many cloudless skies. This observation is verified by a comparison of the curves in Fig. 1 with those in Fig. 2 which are from measurements made at Vienna

[1] Crova, Annales de Chimie et de Physique (6), XX., p. 480.

on a morning of a cloudless June day. It will be noted that curve V_2, Fig. 2, is almost identical with curve V, Fig. 1, and that V_3 differs from these but slightly. All of these curves show the same characteristic, namely, a slight droop in the extreme violet. That the light from a cloud in shadow is ever identical in quality with that from blue skies is, I believe, contrary to the common impression ; yet it is frequently difficult to tell whether a portion of the sky towards which one is looking is clear or clouded unless one can detect the cloud structure.

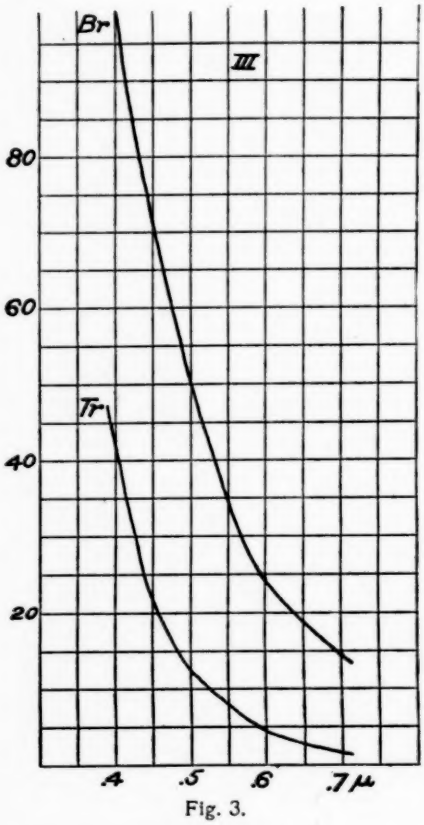

Fig. 3.

While the spectrum of overcast skies is almost identical with that of certain cloudless skies the similarity does not extend by any means to all cases. The intensely blue skies observed in fine weather in regions where the atmosphere is comparatively free from smoke and dust are of quite a different type. Measurements under such conditions give curves of which those in Fig. 3 are good examples. The curve *Br* is from measurements made at Brienz in Switzerland about 8 A. M. on a cloudless morning in August, 1907, while the curve *Tr* was obtained at Trafoi in Tyrol before sunrise in July of that year. In these curves the relative brightness in the violet as compared with the red is several times as great as in the curves for overcast sky or in the curves for clear sky exhibited in Fig. 2. There is moreover no approach to a maximum at the violet end of the spectrum and the yellow and green are relatively very weak so that the curves are strongly concave.

Between these extreme types there exist numerous intermediate stages of the atmosphere which depend on the amount of condensed vapor which may be present. On a misty day, for example, when the sun is barely visible another and distinctive type of curve is obtained. Examples of this form are given in Fig. 4, the curves in which were taken at Zell am See on a misty day during which clouds were continually gathering and disappearing. Curve *a* was taken at 2:30 P. M. at which time no definite cloud forms were visible but the sky was full of white mist. At 2:45 P. M. the mist had cleared and the sky was blue and very bright. Curve *b* represents this condition and it is interesting to compare this curve with those in Fig. 2. The intensities are nearly the same as in V_2. The curve *b* is intermediate in type between V_2 and the curves in Fig. 3 but much closer to V_2 in character. At 3:15 mist again filled the sky and curve *c* was obtained, the type of which corresponds in all respects with curve *a*. The intensities are the same throughout excepting in the extreme violet where measurements are at best more or less uncertain and both curves have the same characteristic maximum in the blue at .42 μ. This maximum is represented in reduced form in the slight droop in the curves from the overcast sky (Fig. 1) and is entirely absent in those obtained from cloudless skies of the type depicted in Fig. 3.

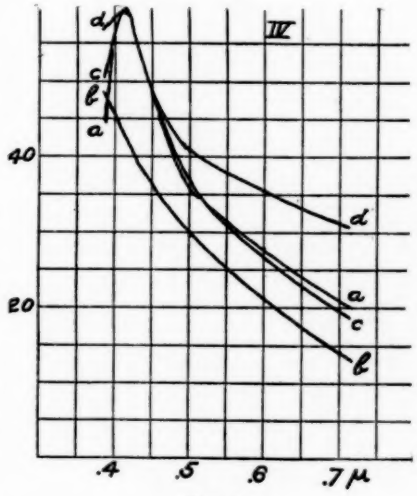

Fig. 4.

At times during the day in question the mists gathered into great masses of sunlit cumulus with patches of clear sky between. At a time when such a cloud mass filled the zenith curve *d* was obtained. In this curve the brightness of the extreme red of the spectrum at .7 μ is more than twice that of the spectrum of the clear sky shown in curve *b*. The intensity in the blue at .42 μ is the same as in

curves *a* and *c* but the peculiar maximum at this region is somewhat less marked.

This maximum in the blue, which is found in all spectrum curves for misty skies which I have observed, appears regularly as the day advances in the mountainous regions of Switzerland during fine summer weather when there is a gradual formation of mist in the upper atmosphere gathering into cloud masses with a tendency toward thunder showers in the afternoon. The change in the curve occurs before the presence of mist is easily observable to the eye. The maximum is commonly well established before noon and frequently persists until towards sunset.

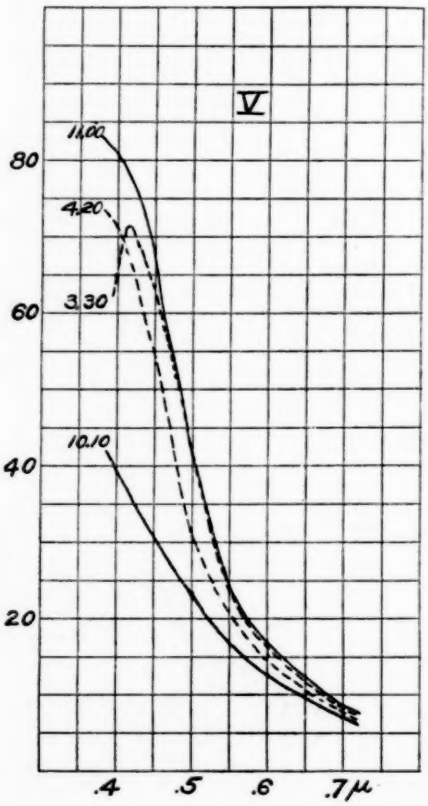

Fig. 5.

The gradual growth and disappearance of this characteristic may be observed whenever observations are made at intervals throughout the day under weather conditions such as I have described. Fig. 5 shows results obtained at the top of the Brienzer Rothhorn in August, 1907. The curve taken at 10:10 A. M. shows no trace of the maximum in the blue. At 11 A. M. the intensities in the blue and violet had rapidly increased and the approach to a maximum in this region began to manifest itself. At 3:30 P. M. the maximum was well marked; at 4:20 P. M. it had again almost disappeared, the type of curve being almost identical with that obtained at 11 A. M. A much more complete series of observations showing these gradual changes of form during the day was obtained at Sterzing on July

17. Measurements were begun before direct sunlight reached the valley and were continued until after sunset.

The general character of the results are shown in Fig. 6 in which the curves drawn in full line are from measurements taken before noon while the dotted curves give data obtained in the afternoon. It will be noticed that the curves at 5 : 20 A. M. and 7 A. M. are of the usual early morning type and similar to those of Fig. 3. At

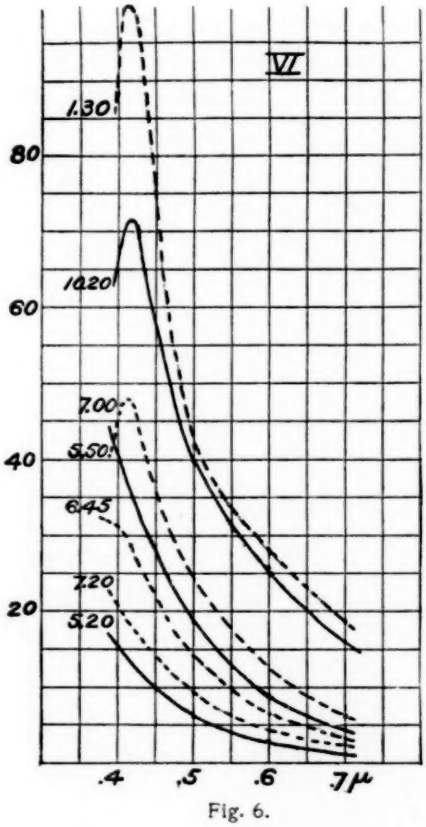

Fig. 6.

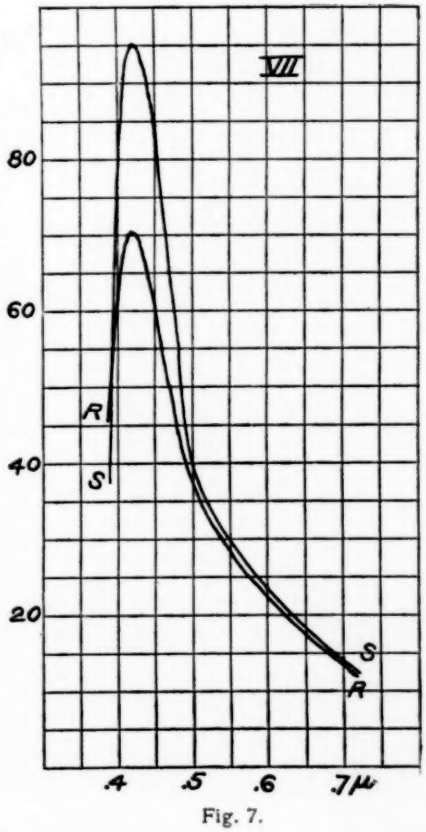

Fig. 7.

10 : 20 A. M., however, the maximum in the blue had developed and it persisted until late in the day. The curves at 1 : 30 P. M. and 5 : 50 P. M. show the continuance of this feature but at 6 : 45 P. M. it had nearly disappeared and at 7 : 20 P. M. was entirely gone. The curve at that hour had returned almost exactly to the form obtained before sunrise.

The maximum in the blue seems to occur in its most pronounced form at high altitudes. The best examples of it which I obtained were observed at Samaden in the upper Engadine and on the upper ice field of the Rhone Glacier.

In curve *S*, Fig. 7, which represents the readings taken at Samaden about 9 o'clock on a morning in July, when the sky appeared quite cloudless, the maximum is developed to an unusual degree. It will be noted that the spectrum of the light from the sky was relatively no brighter, compared with that of the acetylene flame, in the extreme violet at .38 μ than in the green at .50 μ. In the intervening region the ordinates rise however to double these values.

Curve *R* in Fig. 7 is from measurements made on the ice field of the Rhone Glacier at a point above the ice fall near which crossing is usually made from the Belvidere to the Naegeli Graetli. It was a bright morning but mists were continually forming on the surrounding peaks and melting away so as to leave the sky overhead apparently clear. It will be noted that the intensities of the spectrum throughout the region lying between .5 μ and .7 μ are almost identical with those obtained at Samaden. The variations lie almost entirely in the blue and violet of the spectrum. Although in these regions the curves do not coincide, the maximum at .42 μ appears in both.

Whether this highly variable band in the blue is an emission band superimposed upon the spectrum of a sky greatly diluted and whitened by the sunlight reflected from particles of condensed vapor; in which case it might be ascribed to the fluorescence of some variable component of the upper atmosphere, possibly ozone as suggested by Hartley; or whether it is wholly due to selective absorption in the ultra-violet on one side and in the green on the other, as seems much more probable, cannot be definitely determined from existing data. In any event it is a phenomenon absent from the clearest skies and one which is rapidly masked by the presence of considerable thicknesses of cloud-materials. Whatever be its nature it indicates an occasional departure from the distribution of intensities which characterizes the perfect sky of Rayleigh of a sort which demands further investigation.

It will be seen from the foregoing that there are between the

typical curves for the unclouded sky as exhibited in Fig. 3 and the curves for completely overcast sky (Fig. 1) a number of intermediate forms of considerable complexity. The presence of condensed vapor has the effect of increasing the intensity of the longer wave-lengths of the spectrum so that the ordinates of the curves are raised throughout the red, yellow and green. At the same time the maximum in the blue is developed and the spectrum of light from the sky shows the phenomenon of selective reflection to a remarkable degree. Light from sun-illumined cloud masses exhibits the further modifications shown in curve *d* of Fig. 4 in which the intensities in the red, yellow and green reach their highest values. The maximum in the blue is in this case still noticeable although by no means so marked as in the case of light from a clear sky in the presence of incipient mist.

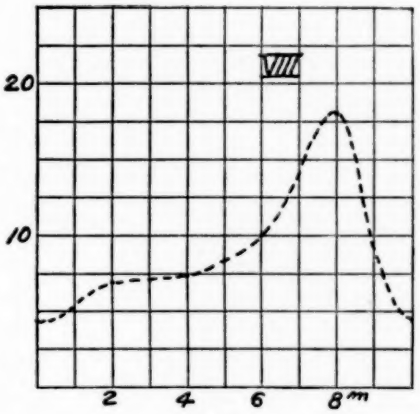

Fig. 8.

That the brightness of the sky increases with the gathering of cloud masses up to the point where sunlight is completely shut out and the sky becomes thickly overcast is clearly shown by means of the results graphically recorded in Fig. 8. The readings from which this curve was plotted were all made from a single wave-length in the red end of the spectrum. The observations were taken during the rapid gathering of clouds upon a showery day, the initial condition being that of a blue sky with no visible mist. Measurements were made as rapidly as possible during the interval of ten minutes at the end of which the sky was completely and heavily overcast. The curve which has time as abscissæ counting from the first observation and intensities of the wave-length in question (.7 μ), in terms of that of the acetylene flame as ordinates, rises to a well-marked maximum after eight minutes and then falls to a value after ten minutes almost identical with the value of the

initial reading. The maximum corresponded as nearly as could be observed to the sudden exclusion of direct sunlight from the masses of cloud under observation and the curve seems to indicate that up to the point where this occurred brightness increased rapidly as the mist gathered. Observations by Basquin[1] and others upon the illumination received from the sky under different conditions are quite in accord with the indications given in this curve. The brightest sky corresponds to a cloudy rather than a clear condition of the heavens but after a certain density of the cloud masses has been attained the illumination falls off in consequence of the exclusion of direct sunlight from the visible surfaces of the clouds.

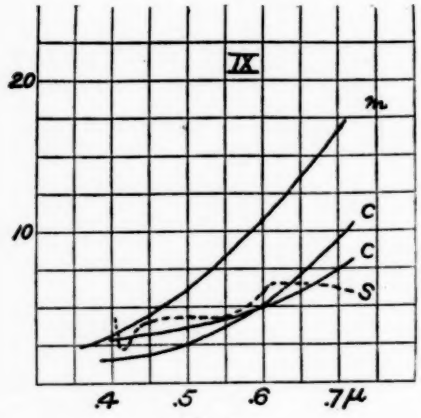

Fig. 9.

In the beginning of this paper I spoke of the curves obtained from overcast skies as being of a simple character by which is meant that there is less evidence of selective reflection than is the case from the open sky. The distribution of intensities is such as to indicate a composition of light closely related to that of radiation from the carbon in the comparison flame although of course of very much higher temperature. Nearly all measurements of light from unclouded skies, particularly at times when the surface of the earth, as well as the upper air, are in sunlight show on the other hand more or less well defined selectivity. A comparison of such curves with a curve for light from the sky taken upon a clear morning before sunrise — what I have called the *typical dawn curve* in the first of my previous papers already cited — shows a more or less complicated relationship like that represented in the curve *s* in Fig. 9. I have discussed the nature of these ratio curves in the paper in question. When, however, we plot similar curves for the ratios of overcast or mist-filled skies to the typical dawn curve, in cases

[1] Basquin, Illuminating Engineer (N. Y.), Vol. I., p. 829.

where the fog is of sufficient density to prevent the direct illumination of the surface of the ground, we get curves of the form *c* and *m* in Fig. 9 which are quite free from the irregularities presented by the curve *s*. This would seem to indicate that the selectivity of ordinary sky-light is due on the one had to light reflected from the surface of the ground to the atmosphere and on the other to selective reflection which takes place in the upper layers of the atmosphere and which is cut out by the intervention of a layer of mist or cloud so that the light from overcast skies is more directly related to sunlight although modified as to the distribution of intensities than is the light commonly observed in the case of unclouded skies.

PHYSICAL LABORATORY OF CORNELL UNIVERSITY,
June, 1908.

NOTE ON THE SHORT-CIRCUITING OF CADMIUM CELLS.

BY P. I. WOLD.

SEVERAL of the cadmium cells mentioned in a previous article [1] have been short-circuited for different lengths of time to find the rate at which they recover and some of the recovery curves obtained are given below.

The period of short-circuiting varied from 15 seconds to 10 minutes and the general shape of the curves is the same in all cases and agrees very well with the curves obtained in a similar manner by F. E. Smith [2] and by Barnett.[3]

The internal resistance of these cells is about 1,500 ohms so that the actual current while short-circuited is rather small. The deviation from the normal value was obtained by a Wolff potentiometer, the short-circuited cell being placed in opposition to a cell taken as a standard and their differences read. Deviations from the normal were read directly to 1/100,000 of a volt and were estimated, with a fair degree of accuracy, to 1/10 of this.

The curves shown are typical curves out of a large number.

Fig. 1 shows the close agreement between two cells, 44 and 45, both short-circuited for 15 seconds. The cell 45 was short-circuited five times for a period of 15 seconds and the recovery curves were identically the same to within the width of a line.

Fig. 2 shows the effect of short-circuiting cell 44 differen tlengths of time. Curve 1 is for 15 seconds, curve 2 for 30 seconds, curve 3 for 60 seconds, curve 4 for 2 minutes, curve 5 for 5 minutes and curve 6 for 10 minutes.

In these two figures the axis OA represents the normal value of the E.M.F. of the cells and the ordinates represent the deviation from this normal in hundred-thousandths of a volt.

[1] PHYSICAL REVIEW, vol. 27, p. 329, 1908.

[2] Philosophical Transactions of the Royal Society of London; Series A, vol. 207, p. 393.

[3] PHYSICAL REVIEW, vol. 18, p. 104, 1904.

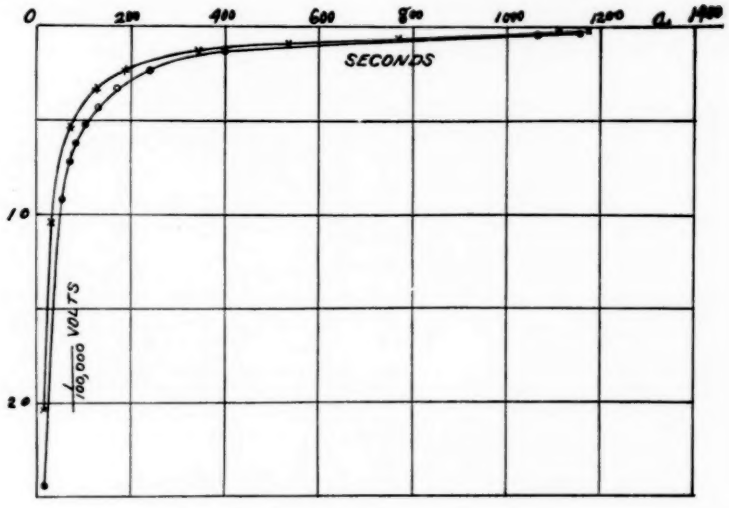

Fig. 1.

In his work Barnett subjected his cells to a much more severe test than in the present case, but apparently in this case the author was able to carry the measurements one place further in the decimal. Even with this increased accuracy in readings, the points, as plotted, fall remarkably close to the curve, indicating, to this degree of re-

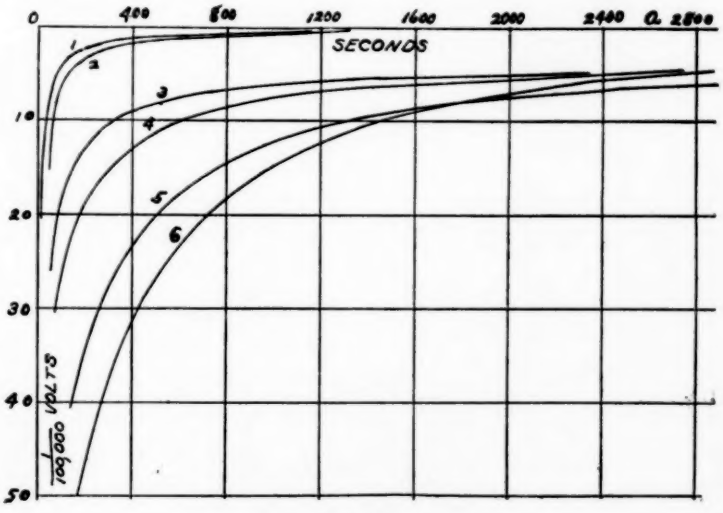

Fig. 2.

finement, a very definite and smooth recovery curve free from accidental disturbances within the cell itself.

An attempt to find a general form of equation for the recovery curve has, so far, been unsuccessful. The form suggests a logarithmic curve, but plotting logarithms of the deviation with time fails to give even an approximately straight line.

The general similarity of the decay curves in the case of phosphorescence, as shown by Professor Nichols and Merritt, with these recovery curves then suggested that these two effects might obey similar laws since in both cases the curve is probably determined by the reunion of free particles; in the one case probably electrons, in the other case the chemical ions which cause the polarization.

In their work on phosphorescence [1] Professors Nichols and Merritt have concluded that the curve obtained by plotting the reciprocal of the square root of the intensity with time is a straight line for small values of t and that it changes to a curve concave to the axis of t as t increases; but for larger values of t the relation between $I^{-\frac{1}{2}}$ and t is again linear, the two straight portions merging into each other.

The results when the values of $D^{-\frac{1}{2}}$ and t, as given in the curves of Fig. 2, were plotted as shown in Fig. 3, the numbers of these curves corresponding to the numbers in Fig. 2. In this case D represents the intensity or amount of polarization, *i. e.*, the deviation of the cell from normal E.M.F. The curves agree in form surprisingly well with the corresponding curves shown by Professors Nichols and Merritt.

Because of the difficulties in obtaining readings in the very first part of the recovery it cannot be said at present that the first part of the above curves would also be straight lines with a different slope. Probably the shape of this curve is due to the fact that there are two, or more, effects giving or intensifying the polarization from which the cell must recover, and these individual recoveries are of different velocities. It may be that one phase of the recovery is brought about by the diffusion of ions, which at first are concentrated close to a separating surface in the cell, and the other part of the recovery is due to the reunion of these ions. At first probably

[1] PHYSICAL REVIEW, vol. 22, p. 279, 1906; vol. 23, p. 37, 1905.

the diffusion effect would be the predominant one, following one law, but later the chief effect would be that due to reunion of ions, which would follow a different law, similar perhaps to the law governing the reunion of electrons in the case of phosphorescence.

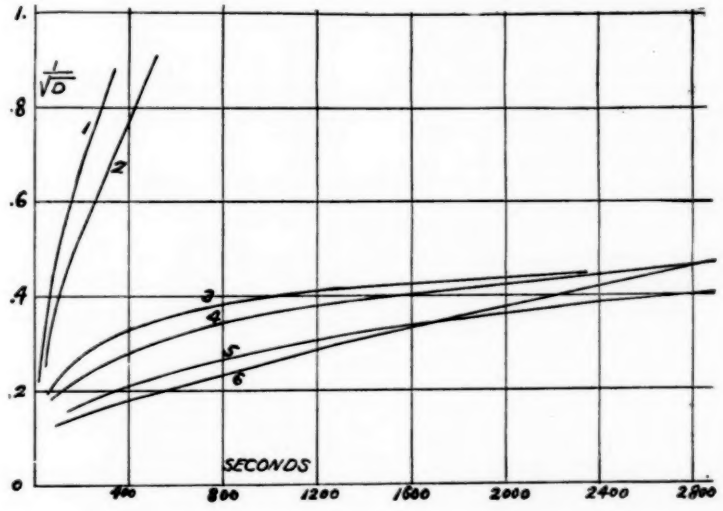

Fig. 3.

It may be, on the other hand, that we have simply the case of an excess of ions, producing polarization, at different points; as SO_4 ions near the cadmium and cadmium ions near or in the $HgSO_4$ and that the rate of recovery from these effects is different.

The author hopes to be able to do more work on this matter with the idea of determining with which case we have to deal.

PHYSICAL LABORATORY,
CORNELL UNIVERSITY.

THE DISPERSION OF ELECTRIC DOUBLE REFRACTION.

BY T. H. HAVELOCK.

A RECENT paper by C. F. Hagenow,[1] on electric double refraction in carbon bisulphide suggests a comparison of the experimental results with a formula which the present writer has given connecting the dispersion of artificial double refraction with the ordinary dispersion of the isotropic medium.[2] In this connection the theory may be indicated briefly in the following manner.

In any medium the total electric displacement consists of two parts: (1) the æthereal displacement, measured by $(4\pi)^{-1}$ times the electric force, (2) the material polarization of the particles constituting the medium. The latter part depends upon the effective electric force acting upon the particles, and this in turn depends for each particle upon the character and arrangement of the surrounding particles. We imagine an effective cavity to be associated with each particle, such that the force on the particle is the force within such a cavity in a medium uniformly polarized to the value of the material polarization vector at the point in question. For an isotropic medium the effective cavity is taken to be a sphere; as is well known, this leads to the formula

$$\frac{n_0^2 - 1}{n_0^2 + 2} = \frac{4\pi}{3} \Sigma_h \frac{a_h}{k_h^2 - k^2}, \qquad (1)$$

where n_0 is the refractive index of the medium, $2\pi/k$ is the period of the vibrations, and a_h, k_h are constants assigned to each type of particle, the summation being extended over the various types. Suppose that the medium is subjected to mechanical stress, or to the action of an electric or magnetic field, so that it becomes doubly refracting; assume further that the consequent slight modification of the optical properties can be represented by a change of the effec-

[1] C. F. Hagenow, PHYSICAL REVIEW, vol. 27, p. 196, 1908.
[2] T. H. Havelock, Proc. Roy. Soc., A., vol. 80, p. 28, 1907.

tive cavity from a sphere to an ellipsoid. Let the axes of the ellipsoid be in the ratios of $1 + \varepsilon_1$, $1 + \varepsilon_2$ and $1 + \varepsilon_3$; also let n_1, n_2, n_3 be the principal refractive indices of the medium.

The components of effective force within the cavity can be evaluated and we obtain, instead of (1), three equations of the type

$$\frac{n_1^2 - 1}{n_1^2 + 2 + 3s_1(n_1^2 - 1)} = \frac{4\pi}{3} \Sigma' \frac{a_k}{k_k^2 - k^2}, \tag{2}$$

where

$$S_1 = -\frac{4}{15}\varepsilon_1 + \frac{2}{15}(\varepsilon_2 + \varepsilon_3).$$

Hence from (1) and (2) we can find expressions for the principal indices in terms of n_0, the refractive index of the isotropic medium composed of the same particles in equal density.

If the strained medium behaves like a uniaxial crystal, we have

$$n_1 = n_0 - \frac{2}{15}\frac{(n_0^2 - 1)^2}{n^0}\varepsilon$$

$$n_2 = n_0 + \frac{1}{15}\frac{(n_0^2 - 1)^2}{n_0}\varepsilon.$$

Hence the dispersion of the double refraction, measured by the difference of the ordinary and extraordinary indices, is given by

$$\delta \propto \frac{(n_0^2 - 1)^2}{n_0} \tag{3}$$

This formula connects the dispersion of the slight artificial double refraction with the ordinary dispersion of the isotropic medium. The relation suggested by Kerr as the result of experiments was, that the double infraction induced in CS_2 by an electric field was inversely proportional to the square root of the wave-length, but this has been disproved by the experiments of Blackwell,[1] and again by those of Hagenow. In the paper already quoted, the writer has shown that the formula (3) agrees well with Blackwell's results; this is also the case for the more recent data, as is shown by the following table. The first two columns give the experimental data which are taken from Hagenow's paper; these are the average results of sets of observations and it is claimed that the experimental error is small,

[1] H. L. Blackwell, Proc. Amer. Acad., vol. 41, p. 650, 1906.

except that the value of δ for λ equal to 600 appears to be abnormally low and the value of λ equal to 700 is also not so reliable. The values for n_0 given in the third column were obtained by the help of the table given by Martens,[1] using also a temperature correction of -9×10^{-4} for each degree rise in temperature; we obtain thus values correct to three decimal places. The fourth column shows the values of $\delta n_0/(n_0^2 - 1)^2$, which ought to be constant by relation (3); omitting the values at 600 and 700 for the reasons stated above, we have a mean value of 60.57 for the remaining six values, showing a maximum divergence from the mean of about .6 per eent.

CS_2 at 25°.

λ	δ obs.	n_0	$\delta n_0/(n_0^2-1)^2$
430	116.2	1.672	60.27
450	112.5	1.662	60.22
500	106.8	1.643	60.75
550	102.1	1.630	60.60
590	100	1.623	60.78
600	97.5	1.621	59.68
650	97.1	1.614	60.83
700	96.7	1.610	61.42

A preliminary account of investigations on the magnetic and electric double refraction of nitrobenzene has recently been published by Cotton and Mouton.[2] This liquid is interesting on account of the double refraction being extremely large compared with that of CS_2; a series of observations is given showing the magnetic double refraction at various wave-lengths and it is stated that electric double refraction of the same order was obtained. Owing to the present lack of more exact data both for the double refraction and for the ordinary dispersion of nitrobenzene under similar conditions it is not possible to compare the formula given above with these results.

As a result of their series of experiments on magnetic double refraction, dealing originally with the Mayorana effect in colloidal solutions, Cotton and Mouton believe that the phenomena are due to orientation of particles, and they suggest a similar explana-

[1] Martens, Annalen der Physik, vol. 6, p. 632, 1901.

[2] Cotton and Mouton, Comptes Rendus, t. 147, p. 193, 1908.

tion for the electric double refraction in nitrobenzene, namely, orientation of the molecules. In the theory sketched above, an essential point is in regarding the effective cavity to be changed from a sphere to an ellipsoid by the impressed field. Now this may be due to an orientation of similar æolotropic particles, if we regard the isotropy of the unstrained medium as an average effect of orientation of the particles in all possible directions; or it may be due to a rearrangement in grouping of an assemblage of similar particles. In deducing the relation (3) given above, the latter hypothesis was used, and we have seen that the formula is supported by the experimental data available at present.

Whether a similar formula could be obtained on the hypothesis of orientation or not, it seems desirable to have as far as possible parallel series of observations on the dispersion of the artificial double refraction and the ordinary dispersion of the isotropic medium taken under similar conditions, for instance as regards temperature and purity of materials.

ARMSTRONG COLLEGE, NEWCASTLE-ON-TYNE,
November 6, 1908.

PROCEEDINGS

OF THE

AMERICAN PHYSICAL SOCIETY.

MINUTES OF THE FORTY-FIFTH MEETING.

A JOINT meeting of the Physical Society and Section B of the American Association for the Advancement of Science was held in the Physical Laboratory of Johns Hopkins University, Baltimore, Md., December 28–31, 1908.

The President of the Physical Society, Edward L. Nichols, and the Vice-President of Section B, Karl E. Guthe, presided at alternate sessions.

At the afternoon session of Tuesday, December 29, the retiring Vice President of Section B, Dayton C. Miller, delivered an address on The Influence of the Material of Musical Instruments upon the Quality of the Sound produced.

At the business meeting of the Physical Society, held Wednesday afternoon, December 30, 1908, the following officers were elected for the year 1909: Henry Crew, President; W. F. Magie, Vice-President; Ernest Merritt, Secretary; J. S. Ames, Treasurer; J. C. McLennan, Ernest F. Nichols, Members of the Council.

The following papers were presented:

Fatigue of Metals Excited by Roentgen Rays. L. T. MOORE and R. E. C. GOWDY.

Errors in Magnetic Testing of Ring Specimens. M. G. LLOYD.

Data regarding Recent Magnetic Storms. L. A. BAUER.

Optical Properties of Electrolytic Films of Iron, Nickel and Cobalt. C. A. SKINNER and A. Q. TOOL.

An Absolute Gauge for Measuring High Hydrostatic Pressures. P. W. BRIDGMAN.

The Resistance of Mercury as a Secondary Gauge for High Pressures. P. W. BRIDGMAN.

Methods for Measuring Compressibilities at High Pressures. P. W. BRIDGMAN.

An Experimental Determination of the Terminal Velocity of Fall of Small Spheres in Air. JOHN ZELENY and L. W. MCKEEHAN.

Note on the Effect of the Phase of Harmonics in Sound Waves. M. G. LLOYD and P. G. AGNEW.

Magnetic Double Refraction Normal to the Field in Liquids. C. A. SKINNER.

The Absorption Spectra of Various Potassium and Uranyl Salts in Solution. HARRY C. JONES and W. W. STRONG.

New Series in the Spectra of Ca, Sr and Ba. F. A. SAUNDERS.

Ionization in Closed Vessels. W. W. STRONG.

Velocity of the Negative Ions Produced by Ultra-violet Rays in Various Gases at Different Pressures and Temperatures. ALOIS F. KOVARIK.

Momentum Effects in the Electrical Discharge. F. E. NIPHER.

Electrical Stimulation of Plant Growth. AMON B. PLOWMAN.

Note on the Kathode Equilibrium of the Weston Cell. F. A. WOLFF.

The Theory of Coupled Circuits. LOUIS COHEN.

Photographic Registration of Sounds. DAYTON C. MILLER.

The Effect of the Magnetic Impurities in the Copper Coils of Moving Coil Galvanometers upon their Sensitiveness, Hysteresis and Zero Shift. ANTHONY ZELENY.

The Three Temperature Coefficients of the Moving Coil Galvanometer and their Relation to the Temperature Coefficients of its Various Parts. ANTHONY ZELENY and O. HOVDA.

A New Method for the Absolute Measurement of Resistance. E. B. ROSA.

A Plea for Terrestrial and Cosmical Physics. L. A. BAUER.

The Ellipticity of the Earth is Not a Proof of a Former Liquid State. JOHN F. HAYFORD.

Atomic Theories. L. T. MORE.

The Dynamophone. J. BURKITT WEBB.

An Electrical Method for Determining the Amount of Moisture in Grain and Other Materials. ANTHONY ZELENY.

On the Extra Transmission of Electric Waves. F. C. BLAKE.

Entladungsstrahlen. ELIZABETH R. LAIRD.

A Spectrometer for Electromagnetic Radiation. A. D. COLE.

A Method of Determining the Electrode Potentials of the Alkali Metals. G. N. LEWIS and C. A. KRAUS.

Non-Newtonian Mechanics and the Principle of Relativity. G. N. LEWIS and R. C. TOLMAN.

On the Influence of Temperature and of Transverse Magnetization upon the Electrical Resistance of Bismuth and Nickel. F. C. BLAKE.

A New Form of Standard Resistance. EDWARD B. ROSA.

A Proposed Modification of the Kirchoff Method for the Absolute Measurement of Resistance. FRANK WENNER.

An Instrument Designed for More Precise Determination of the Magnetic Declination at Sea. W. J. PETERS.

The Electric Conductivity of the Atmosphere over the Pacific Ocean. PAUL R. DIKE.

The Ultra Violet Absorption and Fluorescence and the Complete Balmer Series of Sodium Vapor. R. W. WOOD.

Results of Recent Intercomparisons of Magnetic Standards by the Carnegie Institution. J. A. FLEMING.

Some Optical Effects of Changes in Ether Density. (By title.) CHAS. F. BRUSH.

The Thermodynamics of Saturated Vapors. (By title.) J. E. SIEBEL.

The Heat Balance in Thermoelectric Batteries. (By title.) J. E. SIEBEL.

The Lumeter, a Practical Measure of General Luminosity. (By title.) HENRY E. WETHERILL.

A Critical Review of the Problem of Pressure in the Kinetic Theory of Gases. L. D'AURIA.

Arc and Spark Phenomena in the Secondary of a High Potential Transformer. (By title.) E. S. JOHNNOTT.

On the Diurnal Viarations in the Intensity of the Penetrating Radiation Present at the Surface of the Earth. G. A. CLINE.

On the Character of the Radiation from Potassium. J. C. MCLENNAN.

The Action of Electrolytes on Copper Colloidal Solutions. E. F. BURTON.

The Upper Inversion in the Atmosphere. W. J. HUMPHREYS.

Some Remarks on Solar Magnetism. W. J. HUMPHREYS.

Note on Thermoluminescence. ELIZABETH R. LAIRD.

The society passed a vote of thanks to the authorities of Johns Hopkins University, and especially to the members of the department of physics, for the many courtesies extended to the Society during the meeting.

Adjourned Thursday, December 31, 1908, at 5 P. M.

ERNEST MERRITT,
Secretary.

A HIGH TEMPERATURE REGULATOR.[1]

BY H. M. RANDALL.

THE variation in the temperature of the oven is detected by a thermo-couple or a platinum thermometer. The coil of the galvanometer in the potentiometer or the Wheatstone Bridge circuit is provided with a light aluminum boom, *b*. With rise of temperature this boom moves between two spring jaws, *s*, which open and close upon one another regularly, the movement being caused by the rotation of the cylinder, *C*, provided with the elevations, *p*. *C* is driven by a motor at

[1] Abstract of a paper presented at the Evanston meeting of the Physical Society, November 28, 1908.

low speed. The platinum contact strips, K, are insulated by hard rubber blocks from the steel springs, s, and just escape contact with each other when the boom is not between them. The diameter of C is such that the springs, s, pinch the boom with considerable force when it is between them in the position shown in the figure, thus completing a battery circuit containing an electro-magnet; this latter, responding, shunts

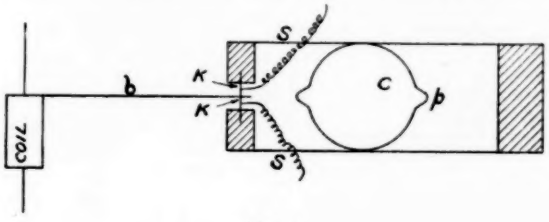

Fig. 1.

part of the heating current of oven. As the oven cools the boom moves from between the jaws through its zero positions to a second similar pair of jaws on the opposite side, which operate to open the shunt circuit. With proper adjustment no strain is put upon the galvanometer suspensions when the jaws close upon the boom: thus light suspensions may be used. With this arrangement it has been possible to maintain for hours temperatures in the neighborhood of 1000 °C. in the oven employed, without any variation in temperature occurring which could be detected by a couple, a Siemans and Halske potentiometer being used.

The Absorption Spectra of Various Potassium and Uranyl Salts in Solution.[1]

By Harry C. Jones and W. W. Strong.

THE methods used in this work are practically those described by Jones and Anderson in publication number 110 of the Carnegie Institution of Washington.

Potassium ferricyanide in aqueous solution was found to obey Beer's law for dilutions ranging between normal and 1/4096 normal.

The limit of absorption was very sharp. Three weak bands at $\lambda\lambda$ 3220, 3000 and 2550 were found.

Potassium ferrocyanide in aqueous solution was found to obey Beer's law between concentrations ½ normal and 1/128 normal. Potassium chromate in water obeys Beer's law between the ranges of concentration 2 normal and 1/2048 normal, potassium dichromate between ⅓ normal and 1/1536 normal.

[1] Abstract of a paper presented at the Baltimore meeting of the Physical Soc ety, December 28–31, 1908.

The uranyl salts investigated have been the nitrate, sulphate, acetate, chloride and bromide. Beer's law was not found to hold except for dilute solutions. Water, methyl and ethyl alcohols have so far been used as solvents. The absorption is greater in each case in the more concentrated solutions (the amount of salt acting as absorbing agent being the same) except in the remarkable case of the acetate in water.

The absorption spectra consist of at most twelve bands in the blue violet region, the number of bands depending upon the amount of uranyl salt in the solution. The positions of the uranyl nitrate bands are all shifted towards the violet with reference to the bands of the other salts. This does not agree with the results given by other workers. Keeping the amount of salt the same and diluting the solution did not cause any change in the positions of the bands.

Dehydrating agents like aluminium chloride and calcium chloride cause the bands to shift towards the red. It is probable that hydration has some influence on this shifting of the bands, since uranyl nitrate possesses much more water of crystallization than any of the other salts.

The wave-lengths of the uranyl bands are different in methyl and ethyl alcohol solutions from what they are in water. This investigation is being carried out with the aid of a grant from the Carnegie Institution of Washington.

IONIZATION IN CLOSED VESSELS.[1]

BY W. W. STRONG.

DURING the past summer work upon ionization in closed electroscopes was continued. Among the electroscopes used was one whose calibration curve is given on page 44 of the July PHYSICAL REVIEW. The rate of leak was found in a cistern as described in that article. The leak September, 1907, was slightly (5 to 10 per cent.) less than in July, 1908, in the same cistern under the same conditions. A daily variation with a maximum at noon was shown for the electroscopes in the open. The above would indicate that for over a year the natural ionization in the electroscope was practically constant.

Experiments were made to test whether direct sunlight affected the ionization current within an electroscope. Curved electrode electroscopes made of glass were used. The glass was wrapped with fine wire so as to destroy any electrical induction on the gold leaf by external bodies. The effects of air currents and radiometric action is very great but by exposing to sunlight for several hours and then darkening the room for an hour, these difficulties can be partly surmounted. The results indicated that direct sunlight does not increase the rate of leak to any considerable amount.

[1] Abstract of a paper presented at the Baltimore meeting of the Physical Society, December 28–31, 1908.

An Absolute Gauge for Measuring High Hydrostatic Pressures.[1]

By P. W. Bridgman.

In practically all forms of gauge for measuring high pressure the weight is determined which when acting on a piston of known cross-section exactly balances the action of pressure on the other end of the piston. This piston moves freely without friction in an outer cylinder, which it fits so accurately that the rate of leak is very slow. The cylinder is subjected to pressure on the interior only, the effect of which is to increase the inner diameter of the cylinder as the pressure increases. The piston is, on the other hand, subjected to pressure on the exterior curved surface, and therefore loses in diameter with increasing pressure. The effect of the simultaneous swelling of the cylinder and contraction of the piston is a widening of the crack between piston and cylinder under increasing pressure, thus increasing the rate of leak and putting a speedy limit to the pressure range accessible to the instrument. The manometer of Amagat is probably the best known form of this gauge. The pressure limit of the gauge in Amagat's hands was about 3000 kgm. per sq. cm.

In the form of high pressure gauge described in this paper, the above mentioned source of leak is obviated by such a disposition of packing that the cylinder in which the freely moving piston plays is subjected to pressure on the exterior as well as on the interior surface. The effect of the application of pressure to the exterior of the cylinder is to decrease the interior diameter with increasing pressure, thus decreasing the width of the crack through which leak occurs. By properly varying the disposition of the packing the rapidity of the leak may be controlled. The rapidity of the leak is further decreased in this type of gauge by decreasing the dimensions below the customary values. The diameter of the piston used in this gauge was only 1.6 mm., as against 6 mm. used by Amagat. With this gauge pressure of 6,800 kgm. per sq. cm. have been measured repeatedly without perceptible leak. Comparison of two such gauges with each other indicates a probable accuracy of at least one tenth per cent. in the absolute pressure measurement over the entire pressure range, with a sensitiveness at the maximum of about 3 kgm. per sq. cm. The correction which must be applied for the elastic deformation of the metal of the gauge amounts at 6,800 kgm. to only one fifteenth per cent.

[1] Abstract of a paper presented at the Baltimore meeting of the Physical Society, Debember 28–31, 1908.

THE RESISTANCE OF MERCURY AS A SECONDARY GAUGE FOR HIGH PRESSURES.[1]

BY P. W. BRIDGMAN.

IN the preceding abstract, an absolute gauge for hydrostatic pressures up to 6,800 kgm. per sq. cm. was described. In practical use it is inconvenient, because of its bulkiness and the time necessary to make a single measurement. The more common forms of secondary gauge in ordinary use, while undoubtedly more convenient, yet show rapidly increasing sources of error as the pressure is pushed to values as high as the above. The chief of these sources of error is the entrance of hysteresis effects. Another convenient form of secondary gauge proposed by de Forest Palmer in 1897 employs the variation of electrical resistance of mercury. Measurements were made by de Forest Palmer to 2,000 kgm. with a probable error of about 1.6 per cent. in the pressure which might be deduced from the resistance measurement.

In this paper the suitability of the mercury gauge has been investigated over a wider pressure range and with somewhat greater accuracy. It was found that the gauge shows no hysteresis and is capable of giving the pressure to 6,800 kgm. with an accuracy of one tenth per cent. The greatest source of error in actual use is the irregular action of the glass capillary containing the mercury. The glass must be seasoned by repeated subjection to pressure before its indications become regular. The glass must be reseasoned if subjected to large changes of temperature. It was found that an easily fusible glass in which the strains left from drawing are small gives the most regular results. The glass employed was Jena tubing No. 3880a, and it is recommended that this tubing be used if this form of gauge should be employed elsewhere. An empirical formula is given connecting the change of resistance in a capillary of the above glass with the pressure correctly to one tenth per cent. over the pressure range 500–6,800 kgm.

From the compressibility of the glass and of the mercury, which will be determined in another paper, the change of specific resistance of mercury is determined from the apparent change in the above glass envelope. The changed specific resistance corrected for the changed density is also determined. Empirical formulas are given connecting these two resistances with the pressure, but no theoretical discussion of the significance of this change has been attempted.

[1] Abstract of a paper presented at the Baltimore meeting of the Physical Society, December 28–31, 1908.

Methods for Measuring Compressibilities at High Pressures.[1]

By P. W. Bridgman.

In this paper, methods are described for measuring the linear compressibility of solids in the form of rods and the cubical compressibility of liquids. The pressure which went to 6,500 kgm. per sq. cm. was measured with the mercury resistance gauge described in a preceding abstract. The motive in collecting the data was merely the bearing that they have on the variation of the specific resistance of mercury. No determination of compressibilities that might be interesting in themselves was attempted.

Two methods were used for determining the linear compressibility of solids; a comparative method for relatively highly compressible solids like glass, and an absolute method for incompressible solids like steel. In the first method a tube of glass fits into the comparison tube of steel. These are kept permanently in contact at one end, while at the other end there is a ring sliding on the glass and in contact with the end of the steel stube. Fluid pressure is applied all over both steel and glass, producing a displacement of the free end of the glass with respect to the steel. This displacement is indicated by the shift of the brass ring, which can be measured after pressure is released. In the second method, the rod of steel or other substance is placed in a heavy cylinder subjected to pressure on the interior. The effect of this interior pressure is to shorten the steel rod, and to simultaneously lengthen the cylinder. The relative change of length of rod and cylinder is indicated by a ring sliding on the rod like the ring on the glass above, while the absolute change of length of the cylinder is measured from the outside. The difference of these two changes of length gives the absolute change of length of the steel rod. By these methods the linear compressibility of two kinds of glass, of aluminum, and of several grades of steel were determined. The compressibility of the glass shows intrinsic irregularities of perhaps five per cent. Large changes in the carbon content of the steel were found to make very slight changes in compressibility. To test the justifiability of the hypothesis that solids are equally compressible in all directions, two specimens of boiler plate were measured, cut along and perpendicular to the direction of rolling. There was no difference of more than the experimental error, one tenth per cent.

The method for measuring the compressibility of mercury was a revival of an old method due to Perkins in 1826 and since allowed to fall into disuse because of the inaccurate results obtained by him. A freely moving piston is forced by fluid pressure into a steel cylinder containing the mercury. The displacement of the piston is indicated by a sliding ring

[1] Abstract of a paper presented at the Baltimore meeting of the Physical Society, December 28–31, 1908.

similar to the above. From this displacement of the piston and the compressibility of the containing vessel of steel, which may be determined as above, it is easy to calculate the volume compressibility of the mercury. The probable error of this determination was estimated to be about one third per cent. The change of compressibility with pressure is less according to this determination than that found by Carnazzi, who worked to 3,000 kgm., and who made the only previous determination with which these results can be compared at more than 500 kgm.

The values found for the cubic compressibility were as follows:

Jena glass No. 3880*a*, 2.17×10^{-6} kgm./cm.^{2},

Jena glass No. 3883, 2.23×10^{-6} kgm./cm.^{2},

Steel varied from 5.16×10^{-7} to 5.30×10^{-7} kgm./cm.,

Aluminum rod, 11.7×10^{-7} kgm./cm.^{2}.

Volume of mercury is connected with pressure by the relation:

$$\Delta V / V_0 = ap + bp^2,$$

where

$$\log a = 4.5681 - 10,$$

$$\log (-b) = 9.2977 - 20.$$

THE JEFFERSON PHYSICAL LABORATORY,
CAMBRIDGE, MASS.

FATIGUE OF METALS EXCITED BY ROENTGEN RAYS.[1]

BY LOUIS T. MORE AND R. E. C. GOWDY.

THE work is a continuation of the results previously obtained in the same subject (Phil. Mag., 1907, and Chicago meeting of the American Association for the Advancement of Science). A new method has been devised for measuring the secondary radiation given off by metals bombarded by X-rays. Previous results have been confirmed and extended.

To account for the secondary radiation, Professor J. J. Thomson has advanced the theory that the X-rays effect a disintegration of the atomic structure of the metal and permit the expulsion of charged corpuscles. Our experiments make this theory doubtful. Iron, lead and copper plates with pure surfaces were used and then the plates were coated with thin films of the lower oxides of the metal and again with films of the higher oxides. The effect of this successive oxidation on the fatigue seems to show that chemical changes of the surface produced by the X-rays with the consequent changes of surface electrified double layers will account for the phenomena observed.

[1] Abstract of a paper presented at the Baltimore meeting of the Physical Society, December 28–31, 1908.

A Method of Determining the Electrode Potentials of the Alkali Metals.[1]

By Gilbert N. Lewis and Charles A. Kraus.

THE electrode potentials of the metals of the alkalies and the alkaline earths, notwithstanding their great importance, have never been determined, because of the extreme reactivity of these metals. The method now adopted, which has proved entirely successful in the case of the sodium e ectrode, consists in measuring the electromotive force between the metal and its dilute amalgam in mercury with an electrolyte consisting of a solution of a salt of the metal in question in liquid ethylamine. The electromotive force so obtained can readily be shown to be independent of the electrolyte and the solvent used. It is therefore the same as would be obtained if the electromotive force between the metal and amalgam could be measured in an aqueous solution. The potential of the amalgam against a normal solution of a salt of the metal in water may, with certain precautions, be measured directly against a normal electrode. Adding the electromotive force so obtained to the electromotive force between the metal and amalgam, gives directly the potential of the metal in a normal solution of its ion in water, the potential of the normal electrode being taken as zero. In the case of sodium, this method has made it possible to determine the electrode potential within a few tenths of a millivolt. The value obtained is about half a volt higher than that which has been previously assumed for the sodium electrode.

Ultra-Violet Absorption and Fluorescence and the Complete Balmer Series of Sodium Vapor.[1]

By R. W. Wood.

THE absorption spectrum of dense sodium vapor, contained in a steel tube 1 meter in length, provided with quartz windows, and heated red hot in a combustion furnace, shows the lines of the principal series (Balmer formula) reversed. But seven lines of this series have been previously observed, the observations having been confined to the emission spectrum.

Employing a small quartz spectograph by Fuess (focus only 15 cm.) twenty four lines were found and measured and indications of the "head" of the series appeared in the plate though it was not resolved into lines.

With the large quartz spectograph of the Bureau of Standards 48 lines were resolved, bringing us within 0.1 of an Ängstrom unit of the theoretical head of the band. The largest number of lines forming a Balmer

[1] Abstract of a paper presented at the Baltimore meeting of the Physical Society, December 28–31, 1908.

series ever observed in the laboratory is twelve in the case of hydrogen (Cornu and Ames). Solar hydrogen (chromosphere spectrum) shows 29 lines. The sodium series is thus nineteen ahead of any Balmer series ever observed, even in celestial sources. Higher resolving power would doubtless resolve more lines, and the only limit appears to be set by the vapor density and the power of the spectroscope. These ultra-violet lines are accompanied on each side by a channelled spectrum, analogous to the channelled spectra observed in the vicinity of the *D* lines, which form the first member of the Balmer series.

It is in the region of the channelled spectra that the interesting results of the fluorescence of the vapor previously described were found. An attempt was accordingly made to ascertain if the ultra-violet channelled spectra exhibited the same phenomenon. Such was found to be the case. Exciting the vapor with the zinc spark, a strong fluorescence was found to be stimulated by the zinc triplet at wave-length 3344–3302. This triplet is in coincidence with the second line of the Balmer series, at the middle of the accompanying channelled spectrum. A strong fluorescence banded spectrum was found extending from the triplet up to wave-lengh 3630, strongest at wave-length 3612, decreasing in intensity on each side of this point. No fluorescence appeared below the zinc triplet. Exciting with the zinc triplet separated by the spectroscope, we find no visible fluorescence. Exciting with visible light we find no trace of the ultra-violet fluorescence, which indicates that the effect is not carried over from one channelled spectrum to the other. Experiments with other sources of ultra-violet light are in progress.

NON-NEWTONIAN MECHANICS AND THE PRINCIPLE OF RELATIVITY.[1]

BY GIFFORD N. LEWIS AND RICHARD C. TOLMAN.

THE laws of Non-Newtonian Mechanics previously derived by one of the authors from the fundamental conservation laws and from a simple assumption in regard to the nature of light, are identical with those which Einstein has obtained from the principal of relativity and the laws of electro-dynamics. In this paper it is shown that the same equations may be obtained without the aid of the electro-magnetic theory from the principle of relativity and the conservation laws.

[1] Abstract of a paper presented at the Baltimore meeting the of Physical Society, December 28–31, 1908.

Photographic Registration of Sounds.[1]

By Dayton C. Miller.

FOR making large scale records, showing the details accurately, of complex sound waves having frequencies ranging from 500 to 10,000, the phonograph and oscillograph methods seem unsuitable. The following direct mechanical method has given satisfactory results.

A small steel cylinder, 1 mm. in diameter, is arranged to receive angular motion, with a minimum of reaction effects, which is proportional to the displacement of a sensitive diaphragm. A minute mirror, with its plane in the axis of the cylinder, reflects light to a special camera, and at a distance of 30 cm. gives waves 15 cm. wide which show great detail. Results were exhibited.

Note on Thermoluminescence.[1]

By Elizabeth R. Laird.

THEORY suggests that in the case of thermoluminescent salts the change producing luminescence goes on very slowly at ordinary temperatures and is merely accelerated by raising the temperature. An additive method for obtaining the effect of thermoluminescence should therefore show its existence at room temperatures.

This was tested by wrapping up photographic plates for different periods of time with the sensitive film towards some thermoluminescent salt and developing later, at the same time heating the salt to observe the remaining thermoluminescence.

The salts used were a solid salt solution of calcium sulphate and manganese sulphate, the same with an undetermined admixture due to which the thermoluminescence was brighter in patches and was restored in these patches after heating by exposure to light, and calcium sulphide which had been kept in the dark some time after all visible luminescence had ceased.

The results showed that in each case the photographic plate was affected, that where the thermoluminescence was strong as was the case with the second and third salts used if they had been exposed in strong daylight, the effect was obtained in less than two weeks or even in a few days, that when the thermoluminescence was faint as when calcium sulphide was exposed only to dim light a period of two months was required for a marked effect, and that this was also the case with the first named salt solution. Where the effect was uneven the greater effect corresponded to the portions showing brighter thermoluminescence.

[1] Abstract of a paper presented at the Baltimore meeting of the Physical Society, December 28–31, 1908.

Control plates used with salt that had not been exposed to light after heating showed in the same time no effect.

As far as these experiments go they indicate that there is a slow change in thermoluminescent salts, probably similar to that occurring at the higher temperature, and that the total change depends on the original intensity, the time and the nature of the salt.

NEW SERIES IN THE SPECTRA OF CA, SR AND BA.[1]

BY F. A. SAUNDERS.

PHOTOGRAPHS with a quartz spectrograph show a new series of lines in the arc spectrum of Ca whose approximate wave-lengths are 2399, 2276, 2201, 2151, 2118, 2098, 2084, 2073. A similar series is present in Sr, also at λ 2428, 2354, 2309, 2277, 2254, 2239, 2228. In Ba the following lines seem to form the beginning of a similar series, though it fades off very rapidly: 3071, 2597, 2380. The Ca and Sr series are well represented by formulæ only slightly modified from the Ritz type, involving four constants, but no suitable 3-constant formula has been found for them.

The series of pairs in the Sr and Ba spectra whose existence was first announced by Rydberg from the Zeeman effect, and whose formulæ are given by Ritz[2] calculated from the first two members of each subordinate series and the first principal series member, have been extended by the additions of the following new pairs:

Sr.	1st subordinate series, $1/\lambda =$	50122	and	50900
	2d subordinate series, $1/\lambda =$	48717	and	49508
Ba.	1st subordinate series, $1/\lambda =$	44719.1 44771.4 46410.4	and	—— 48676 50326
	2d subordinate series, $1/\lambda =$	43727.5 45418.6	and	48027 ——

A three-constant formula of the Rydberg type suitably represents these series, as enlarged, but the one given by Ritz fails to do so.

[1] Abstract of a paper presented at the Baltimore meeting of the Physical Society, December 28–31, 1908.

[2] Phys. Zeitschr., 9,521, 1908.